A PRICING MODEL FOR FORAGE IN BRITISH COLUMBIA

by

TRENTON JOHN HAGGARD

B.A., The University of British Columbia, 1992

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE

in

THE FACULTY OF GRADUATE STUDIES

(Department of Agricultural Economics)

We accept this thesis as conforming
to the required standard

....

THE UNIVERSITY OF BRITISH COLUMBIA

September 1994

ABSTRACT

The production of forage in British Columbia plays and integral role in sustaining livestock herds within the province. Forage is an important component in the daily feed requirements of horses, sheep, and cattle. Fluctuations in the availability of forage due to drought or bad weather conditions can impose considerable costs on farmers who raise livestock. Wide-spread drought conditions can significantly limit the availability of forage crops within certain regions, causing prices within those regions to become inflated.

Under standard insurance in British Columbia, farmers are only insured against shortfalls in production; there is no compensation provided against increases in the price of forage. For those purchasing forage, a Wide-Spread Drought (WSD) insurance scheme would provide insurance against the price-risk associated with drastic weather conditions. However, since forage prices are required to operate such a policy and are non-observable, a mechanism is needed in order to estimate them. A regional spatial price-equilibrium model which relates regional prices to regional production is developed in this thesis. The model will eventually be used to predict prices and hence determine whether a particular region is eligible for a payout under the WSD insurance scheme. A key assumption behind the model is that according to the 'Law of One Price', prices are perfectly arbitraged. In a competitive setting, in which agents maximize individual welfare, total welfare is maximized and prices between regions will not differ by more than the transportation costs.

This spatial price-equilibrium model is applied to British Columbia forage production. The regions incorporated in the study include the Peace River, Central Interior, Cariboo-Chilcotin, Thompson-Okanagan, and Kootenay Regions. The Lower Mainland/Fraser Valley and Vancouver Island are excluded as they do not typically fall under the forage crop insurance plan in British Columbia.

Table of Contents

List of Figures

List of Tables

Acknowledgements

I would like to thank the members of my committee: Dr. James Vercammen, Dr. Casey van Kooten, and Dr. W. Waters. A special thanks to Jim, my primary advisor, for all of his help and effort towards this thesis. Thanks also to the the guys at the Ministry of Agriculture, Fisheries, and Foods (Crop Insurance Branch): Richard Scott, Lonie Stewart, Wayne Lohr, and Bob France. Without the funding and support from the Ministry and its members, this study would not have been possible.

Thanks to all my fellow students and members of the Department of Agricultural Economics, for putting up with me and providing occasional insight to the thesis. A special thanks to Jodie for the use of her broad vocabulary and superior editing skills.

Mostly, I would like to thank my family for their never-ending support and encouragement. Yeh, this thing is finally done...

Chapter 1 : Introduction

1.1 Background

Forage is an important crop in British Columbia as it contributes to the British Columbia livestock industry. Livestock producers use forage as one of the main ingredients in the composition of livestock feed. Forage in some cases comprises a significantly large portion of livestock dry matter intake (up to 70% for sheep and 100% for horses). Percentage of forage use per dry matter intake for cattle differs depending on type and sex of animal; however, a rough estimate of forage consumption by cattle would be approximately 30 lbs per day, 6 lbs for sheep, and 16-20 lbs for horses.[1]

Relative to the land devoted to its production, forage is a highly significant crop in British Columbia.[2] More than 800,000 acres was devoted to its growth in 1991, compared to slightly over 100,000 acres for each of wheat, barley, and canola. The largest allocation of land devoted to forage occurs in the Thompson-Okanagan Region under which approximately 50 percent of the land is irrigated. Irrigated land can also be found in the Kootenay (approximately 50% of the land), Cariboo-Chilcotin (less than 50%), and Central Interior regions (under 20%), with no irrigation occurring in the Peace River Region (Statistics Canada #95-393D, 1991).

On average, the province tends to produce enough forage to meet its own needs. In a typical year in which average yields occur, both the Peace River and Kootenay Region are relatively self-sufficient (producing enough to meet their requirements). The Cariboo-Chilcotin Region produces less than it requires and, as a result, crops will flow in from the Central Interior and Thompson-Okanagan regions.[3]

---

[1] For further information refer to Keay (1991), National Research Council (1989), Agriculture Canada (1986), and Beames et al. (1994).

[2] Forage refers to alfalfa and other types of hay used as a component in livestock feed.

[3] On average, these two regions produce more forage than they require.

In British Columbia, the majority of forage crops are produced by those who utilize it (for livestock feed). These farmers commonly store some of their crops for use in following seasons. However, in times of drought (periods of unusually low levels of precipitation), this is often not enough to meet their herd's forage requirements. As a result, farmers will purchase forage locally or from other regions (other parts of the province, Alberta, or Washington) at a price which is based on regional supplies and demand.[4] Given that forage has no close substitutes in feed use and is relatively expensive to transport between regions, forage demand tends to be quite inelastic. This inelastic demand, combined with highly variable yields and quality (due to variability in moisture and natural inputs in production), results in a price of forage that is rather volatile.[5]

Since the majority of livestock producers in British Columbia are both producers and consumers of the crop, they are not only concerned when forage prices fall, but when they rise as well. Higher prices often relate to shortfalls in yield, and as a result, farmers must face both a reduced availability of forage and an inflated price in making up the shortfall. A fall in the price of forage is often associated with an excess supply, and given that farmers typically consume the crops they produce, the situation is not as severe. Forage prices vary inversely with yield levels[6] and because wide-spread droughts can occur as frequent as one in five years[7], price-risk is an important consideration.

---

[4] Forage is commonly transported via truck, as it is the most available and convenient method of transport.

[5] It is not uncommon for forage prices to rise 50 percent above the average during a wide-spread drought.

[6] For example, a 20 percent decrease in yields below the average would result in a 20 percent increase in forage prices.

[7] Obtained from yield series data from Agriculture Canada (1970-74), Tingle (1975-87), Forage Cultivar Trial Summary (1980-93).

1.2 Problem Statement

Under standard crop insurance in British Columbia, farmers are only insured/compensated for shortfalls in production that fall below a guarantee level (some percentage of average production), with the losses valued at an average price level. This means that compensation or indemnity payments equal the shortfall in production below a guarantee level multiplied by the average price level. Farmers who must purchase forage are not well covered in a shortfall year because forage prices tend to rise in shortfall years. It has been proposed by the British Columbia Ministry of Agriculture, Fisheries, and Foods that a WSD insurance scheme be designed to address this price-risk facing farmers.

Under a WSD insurance scheme, farmers would be insured against the rise in price of forage due to wide-spread drought, with indemnity payments equalling the shortfall in production below a guarantee level multiplied by the difference between the market price of forage and a price trigger if the former exceeds the latter.[8] A wide-spread drought would be necessary but not sufficient to trigger a payment from this scheme. This is because when adequate stocks are available in nearby regions, stocks would flow in to alleviate the shortage and the price in the shortfall region would not rise above the trigger.

To qualify for a WSD payment, farmers would have to be eligible for standard insurance (i.e., if their actual production falls below the guarantee level) and have regional forage prices exceeding a threshold price level (i.e., exceeding some percentage of the insured value/average price). There is, however, a problem with implementing a policy such as this one, as some measurement of the actual forage price is required. Since no formal market exists for forage crops, prices are non-observable.[9] This means that without some mechanism for determining the actual price levels, the values of indemnities

---

[8] The price trigger level would be some percentage above the average price.

[9] Transactions regarding the sale of forage occur privately between farmers, and prices vary depending on factors such as quality of hay, transportation costs, and types of transactions (personal discounts between friends, bartering, etc).

under this program would be unknown.

The purpose of this thesis is to present a pricing model for forage. Once constructed, this model can then be used for the purposes of crop insurance, as a mechanism will be available for estimating forage prices. Specifically, current levels of forage supplies would be incorporated with the regression results from the study to generate regional forage price estimates. Given these price estimates and the observed production levels, the insurers can determine the level of indemnity payments under the Wide-Spread Drought insurance scheme. Figure 1.1 represents a region for which an indemnity payment will occur. The actual (estimated) price exceeds the price trigger level and production falls below the guarantee level. The level of indemnity is shown by the shaded rectangle.

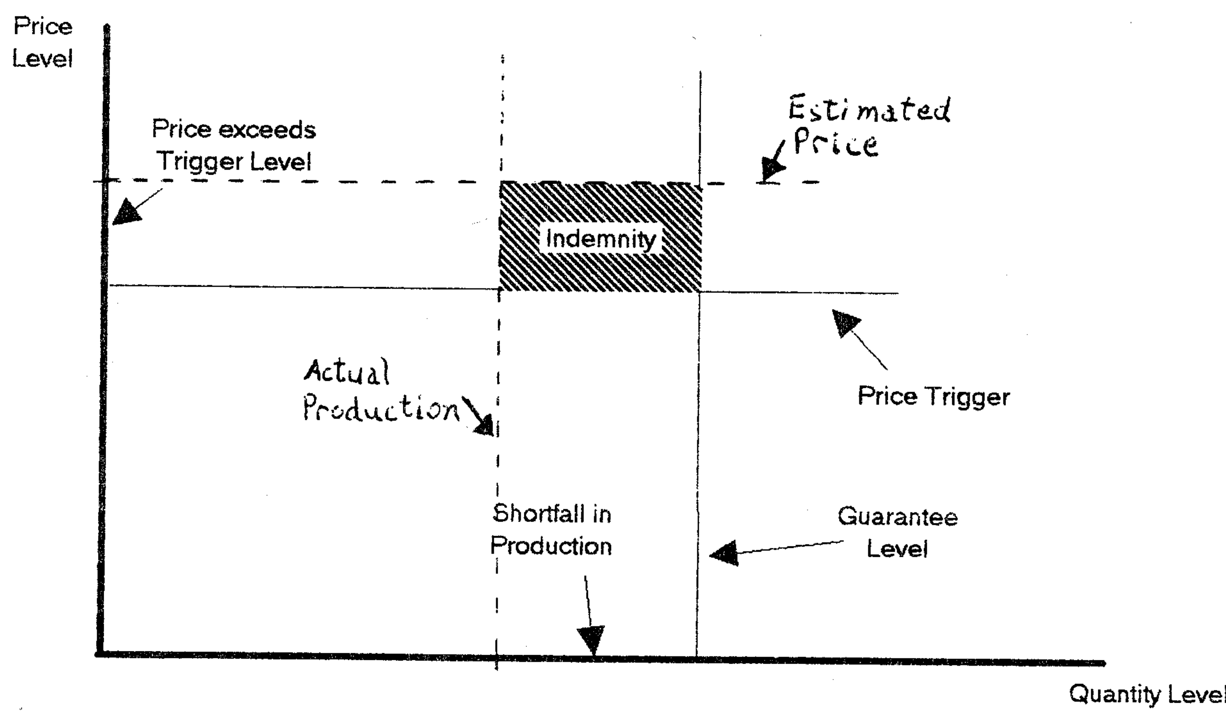

Figure 1.1 Wide-Spread Drought compensation payment.

1.3 Study Objectives

The main purpose of this study is to devise a theoretically acceptable and potentially useful method of estimating the price of forage. Given the problem of trying to establish prices in different areas for a good that flows within and between these areas, incorporating spatial dimensions into the model is required. A regional rather than individual agent model is used because the WSD insurance scheme will be based on regional production and not farm-level production.

Since the market for forage is assumed to be competitive and as a result prices in the regions will not exceed the transportation costs between them, the model utilizes the 'Law of One Price' assumption. The fact that farmers are rational, profit maximizers and crops are able to flow freely between regions ensures that the 'Law of One Price' will hold. The characteristics of each region will be based on representative agents within that region. Furthermore, the ability of each region to place production in storage for use in future periods will also be incorporated.

There are two main components to the model, the first one being that given a set of observations on regional supplies of forage, it will show the equilibrium allocation for those quantities and the set of equilibrium prices. The second component allows for simulations to be run; that is, randomly drawn production levels can be made. The random draws explicitly account for the different production variances and covariances across regions. Combining both components, production levels are drawn and equilibrium allocation levels and prices solved for. This can be done many times in order to get a series of equilibrium prices associated with the simulated quantities. Regression analysis is used to draw relationships between regional prices and quantities. These results can then be used in an insurance scheme, where, given regional quantities of forage, prices can be forecasted.[10]

---

[10] Although econometrics are used in this study, the model presented is a simulation model and not an econometric one. The econometrics are done on simulated and not real data.

1.4 Organization of the Study

Chapter 2 provides a review of the literature and is followed in Chapter 3 by the methodology used in this study. Chapter 3 continues with a description of the model and the assumptions held. Chapter 4 presents an application of the model to British Columbia forage production and includes some description of the data used and its sources. Chapter 5 and 6 follow up with a description of the results, conclusions, and recommendations regarding the model's application. The Appendix contains a description of the data used and generated, sensitivity analysis results, and copies of the computer algorithms used.

Chapter 2 : Review of Literature

2.0 Summary

Assuming that the model would be of a regional, spatial allocation nature, a search of the literature was undertaken. The main focus of the search was to identify past literature that had approached the problem of estimating forage prices (or of similar crops), and which had simulated a spatial allocation type setting.

Prior to 1984, no published studies relating to the estimation of forage prices could be found. A study by Blake and Clevenger (1984) noted the same, and found only one unpublished study by Myer and Yanagida (1981) relating to this topic.[11] Blake and Clevenger stated that the Myer and Yanagida's paper combined an estimated demand function for alfalfa in 11 western states with a quarterly ARIMA model to forecast quarterly alfalfa hay prices. The Blake and Clevenger paper, however, developed a slightly different model that forecasted monthly alfalfa hay prices before the first harvest, for the state of New Mexico. They used a two step procedure that linked an annual model, forecasting the point at which seasonal price patterns start, to a monthly model that identified the seasonal price patterns. They incorporated the estimation of a series of monthly autoregressive price forecasting equations, an annual alfalfa demand equation, and an annual autoregressive acreage forecasting equation. These results were then used to predict monthly alfalfa prices for the state.

In 1987, Blank and Ayer created an econometric model of the alfalfa market for the state of Arizona. A similar study by Konyar and Knapp (1988) provides an analysis for the aggregate California market. A later study by Konyar and Knapp (1990) incorporating much of their previous research, presents a dynamic spatial price-equilibrium model of the California alfalfa market. Their model was used to forecast alfalfa acreage, prices paid and received, and transportation flows for the short and long run under base year conditions. The

---

[11] This Myer and Yanagida study was later published in 1984.

base year results were then used for comparison in determining the effects of reductions in federal water subsidies and the implementation of a cotton acreage-reduction program.

There are many other studies, aside from those focusing on price estimation of agricultural crops, which have focused on spatial allocation and pricing under the spatial allocation setting. One common assumption made in many of these studies is that in a competitive, spatial environment in which goods can move freely from one agent to the next, the 'Law of One Price' holds. There are some studies that may lead one to question the appropriateness of the 'Law of One Price' hypothesis, such as Ardeni (1989) which showed that some of the evidence to support the existence of perfectly arbitraged commodity process in the long run, is flawed due to inferior use of econometric techniques. Other studies counter these attacks, like Baffes (1991) who states that the 'Law of One Price' still holds and any contrary evidence relates accounting for the transaction costs as the failure. Regardless, the 'Law of One Price' hypothesis will be maintained within the current study.

A competitive spatial equilibrium setting is simulated in the study by Liew and Shim (1978). They take the theoretical problem of maximizing an arbitrary net welfare function. It is reduced to the Dantzig-Cottle fundamental problem, which is less complicated than the simplex tableau method as additional variables outside of the original problem are not necessary for obtaining feasible solutions. They discuss the economic implications of dual, slack and surplus vectors and the welfare maximizing marginal transformation of demand and supply among regions. A similar concept to 'The Law of One Price' is assumed, in which the price of the kth commodity in region j should not exceed the sum of the transportation costs required to deliver that commodity from region i to j and the supply price of the kth commodity in region i. A numerical example of the model is also provided.

A paper by Willett (1983) incorporates a typical competitive spatial price-equilibrium model, with both a one commodity and multi-commodity setting represented. This is all done within a linear programming framework. The study further examines and tests the theoretical conditions on prices and quantities, within 'Duality Theory', for a competitive spatial equilibrium solution to be obtained.

A study by Beckmann (1985) offers an interesting look at competitive spatial pricing under two separate pricing techniques. The effects of changing transportation costs, size of fixed costs, and consumer density on the radius of markets, under both techniques is examined. Further, the effects on agents' profits and welfare were examined. Unfortunately, the majority of information provided within this study does not relate directly to the problem of establishing a competitive spatial price-equilibrium type model. Only specific effects that changes in parameters have on the overall solutions are identified.

In Takayama and Labys' (1986) study, a general overview of analysis within a spatial allocation environment is presented. An example of a typical international spatial equilibrium analysis between two countries with one commodity is shown, followed by the general description of a typical interregional spatial equilibrium model. A comparison between the use of the quadratic programming method and linear complementarity programming method in a static spatial equilibrium framework is then made.[12] Furthermore, reference is made to some of the recent models constructed for agriculture, energy, and minerals use. These basically describe the new techniques used by some of the main agents within these sectors.

Another general overview of spatial economic theory is presented in the book by Harris and Nadji (1987). It begins with a general description of 'Spatial Theory', then refers to spatial equilibrium models in relation to 'Location Theory'. It explains that many of the spatial equilibrium models are partial equilibrium special cases of the general theory. The general system as a non-equilibrium dynamic theory is described. Finally, a discussion of the transition of the theoretical framework to an applied model is presented, including a description of the construction of and equations associated with an

[12] A dynamic type framework was not presented in this study.

applied location theory model.

Although there have been few studies done on the pricing of forage (or similar products), much literature exists on the creation of competitive spatial price-equilibrium models. The techniques used in the majority of these studies are directly applicable to the current paper. It was established from the beginning that a regional, competitive, spatial price-equilibrium model was to be used, and the studies shown, provided a general basis for the model represented in this study.

The current study's model typically assumes regions to be both producers and consumers of forage, and allows for forage crops to flow freely within and between regions depending on transport costs, availability, and regional forage requirements. It is a result of this competitive setting that the 'Law of One Price' assumption can be made. The current model is similar to the other spatial price-equilibrium models, especially the one used in Konyar and Knapp (1990). They assume a competitive market exists for the good and that it can flow freely between agents depending on supply and demand. Like the Konyar and Knapp model, the current one assumes regions to be both producers and consumers of the crop. There are, however, a few notable differences that will be outlined.

A dynamic model is used in the Konyar and Knapp study, in which there is a direct link between individual periods. The Konyar and Knapp model contains an acreage response characteristic, where major producing regions have fluctuating acreage depending on acreage from previous periods, expected prices received, and yields. The current study does not make any reference to acreage response, as individual agents are assumed to be price takers and produce forage, independent of expected prices. Making no direct link between periods and having no acreage response characteristic, the current model is not truly dynamic. Carry-over stocks are included in the study but each period is treated as independent of the others.

The Konyar and Knapp study, like the majority of spatial price-equilibrium studies assumes linear inverse demand curves for the good in question, for simplicity. This may be a model mispecification when modelling forage production. Since farmers have specific base feed requirements to meet, and there are costs to adjusting herd sizes, the individual farmers' demand for forage will not fluctuate given small price variability around an average price level. Therefore, an inelastic portion to the demand curve is needed around the average price level when modelling individual farmers. A regional demand curve, however, will not necessarily have the immediate upper and lower kink in the demand and may be slightly smoothed.[13] Nonetheless, kinked regional demand curves were incorporated into the current study, capturing the reluctance of farmers to alter their base herd size.

The current study, unlike the others, uses a simulation and not an econometric model. Production data is randomly generated, with the optimal spatial allocation of supplies and their associated prices calculated. Econometrics is then used on these results in order to formulate a pricing mechanism. In the Konyar and Knapp study, actual production data are used. They also use econometrics in creating the pricing model, however, their results are not simulation-generated as in the current study. Further, the Konyar and Knapp model does not exploit the covariability between regional production (due to common weather patterns).

The aforementioned characteristics of the current study provide some potentially useful techniques which can be added to the past body of research devoted to pricing forage. Studies relating to this topic are few and information regarding the prices of forage can benefit those involved in the production of it and those involved in providing crop insurance and other types of government assistance.

[13] When the representation of forage production is more diverse within a region, the regional demand curve may contain a smoother upper and lower kink.

Chapter 3 : Methodology and Model

3.0 Overview

This chapter introduces a pricing model for forage. The model can be broken down into two main parts: one part generates random regional production and finds the optimal allocation of that production, and the second part performs this over numerous simulations so that a series of quantities and their associated prices can be created. Regression analysis is then used to draw relationships between the quantities and the prices. A more detailed description of these parts is described below.

The first portion of the model allows for regional production levels to be simulated. This is done by randomly drawing regional production levels from normal distributions around their means.[14] Carry-over stocks are added to the randomly drawn production levels to create regional supplies. Given these supply levels, the model optimally allocates quantities for an equilibrium solution. This solution is reached by assuming that each region maximizes its welfare given its own demand and supply for forage, prices in other regions, and transportation costs. As a result, crops will flow within and between regions in order that total welfare be maximized.[15]

The second part of the model randomly draws production levels and solves for equilibrium solutions over a number of simulations. The random quantity levels and their associated prices (as determined by points on the regional demand curves) are collected from each simulation such that a series of quantities and prices are generated.[16] Econometrics is then used to parameterize the relationship between regional quantities of forage and the

---

[14] Normal distributions are used in the random draws, since the associated data requirements are small and multivariate random normal draws are more easily obtained than those with other distributions.

[15] Total welfare equals the sum of each region's welfare.

[16] Prices are also considered to be normally distributed.

associated regional prices.

3.1 Methodology

The model analyzes transportation flows and price fluctuations at a regional level. Each region is described as being both a producer and consumer of forage, and is characterized by a representative agent in that region. It is assumed that the representative farmer produces forage in order to feed his/her own base livestock herd. The regional livestock feed requirements depend on the number of livestock present and the feed requirements per animal.

From figure 3.1 shown below, a typical region has both a production and consumption sector and, depending on current supplies and demand has a number of options in order to meet feed requirements and maximize welfare. If surplus crops are present, the region can allocate stocks to storage for future use or ship to other regions. In times of excess demand, stocks can be drawn from storage (if they are present) or shipped in from other regions. The arrows show the direction in which forage crops will flow.

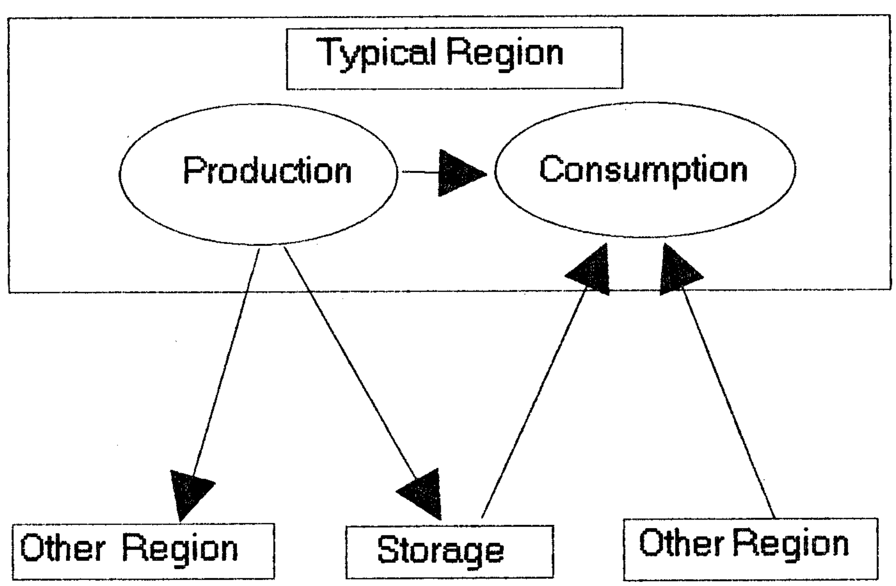

Figure 3.1 Description of a region

Farmers are assumed to be rational profit maximizers. They have a specific base herd size, and will produce forage in order to meet the feed requirements. When a shortfall in production occurs such that the base requirement cannot be met, the farmer will consider purchasing forage to make up the shortfall. The reverse happens when there are surplus stocks. If the base requirement is met, the excess stocks will either be placed into storage for future use or be sold. Only if the price rises to a sufficiently high level or falls to a sufficiently low level will the farmer move away from his/her base herd requirements.

Farm level behaviour must be assumed, since this is a regional model and does not explicitly observe farm level actions. Farmers are considered to be profit maximizers, facing parametric prices. They buy and sell forage in order to maximize their individual profits and as a result, a competitive market is established where total welfare is maximized. Since all farmers are maximizing individual profits (welfare), and total welfare is defined as the sum of all individual welfares, total welfare is maximized. This assumption of welfare maximization allows for the creation of an equilibrium setting in which the model's key assumption, the 'Law of One Price', may hold.

The 'Law of One Price' states that the price in any one region will never exceed the price in another region by more than the transportation costs. It is the assumption of a competitive market that validates the 'Law of One Price' in this model. In a competitive world, individual agents seek to maximize their own welfare, and the collective action of all agents can and will affect prices. Given an arbitrarily high price in one region (i.e., price exceeding that of another region by more than the costs of transportation), individual agents (and as a result the collective of agents) will arbitrage on this high price. As a result, the price will fall until arbitrage is no longer feasible and the 'Law of One Price' holds. Therefore, it can be stated that when individuals maximize welfare the 'Law of One Price' holds, and when the 'Law of One Price' holds, total welfare is being maximized. If the 'Law of One Price' is not holding, then individuals are not profit maximizing and total welfare is not being maximized.

The following diagram, Figure 3.2, presents examples which help to validate that the 'Law of One Price' will hold when welfare is maximized. The first example occurs at point A for both of the regions. In this case, no trade takes place, with Region 1 producing and consuming at the point CA1 and Region 2 producing and consuming at point CA2. Assume that the 'Law of One Price' is violated, where the price in Region 1, PA1, exceeds the price in Region 2, PA2, by more than the costs of transportation. Since an autarky example is being represented, total welfare can be determined by strictly looking at consumption levels under the value of marginal product curves (demand curves). Region 1 consumes at CA1, therefore its welfare can be measured by the area under its curve, areas 1, 2, and 3. Region 2 consumes at CA2, therefore its welfare is shown by areas 7, 8, 9, 10, and 11. The total welfare is measured as the sum of these two welfares.

When the regions are able to trade, crops will flow from Region 2 to Region 1, since the agents can be made better off by this. Trade will take place until the point at which the 'Law of One Price' is no longer violated (the point where the price in region 1, PB1, exactly exceeds the price in 2, PB2, by the transportation costs). It is at this point that trade can no longer make both regions better off, since the price for which the crop is sold is equivalent to its marginal value in consumption.[17] It is at this point that total welfare is maximized, since any other allocation of crops other than this equilibrium allocation will cause total welfare to decrease.

The total welfare associated with the second case has to include both the value from sale and from consumption, since trade has occurred. Region 1 purchases forage from Region 2 and increases its consumption to the point CB1. The cost of that purchase is equal to the price paid, PB1, multiplied by the quantity difference between CA1 and CB1. This is a cost shown by area 5. However, Region 1 now benefits from areas 4 and 5, and therefore, gains area 4. The sale of forage from Region 2 to Region 1 means that 2 now consumes at the lower level of CB2, and therefore, loses the consumption welfare shown by areas

[17] Note that when crops flow from Region 2 to 1, the price in 1 will fall (as supply increases) and the price in 2 will rise (as supply decreases).

10 and 11. However, the sale of forage benefits Region 2 by the value of PB2 (the price sold at) multiplied by the difference between CA2 and CB2. As a result, Region 2 gains a welfare amount equal to the shaded area above the demand curve. For a linear curve this welfare amount is equivalent to area 10.

When the two regions are able to trade, the allocation of crops between the regions will be such that total welfare will be maximized, and the 'Law of One Price' will hold. In this example, both regions are made better of by trading to the point that prices no longer differ by the costs of transportation. Compared with the first example, Region 1 shows an increase in welfare equivalent to area 4, and Region 2 gains an amount equivalent to area 10.

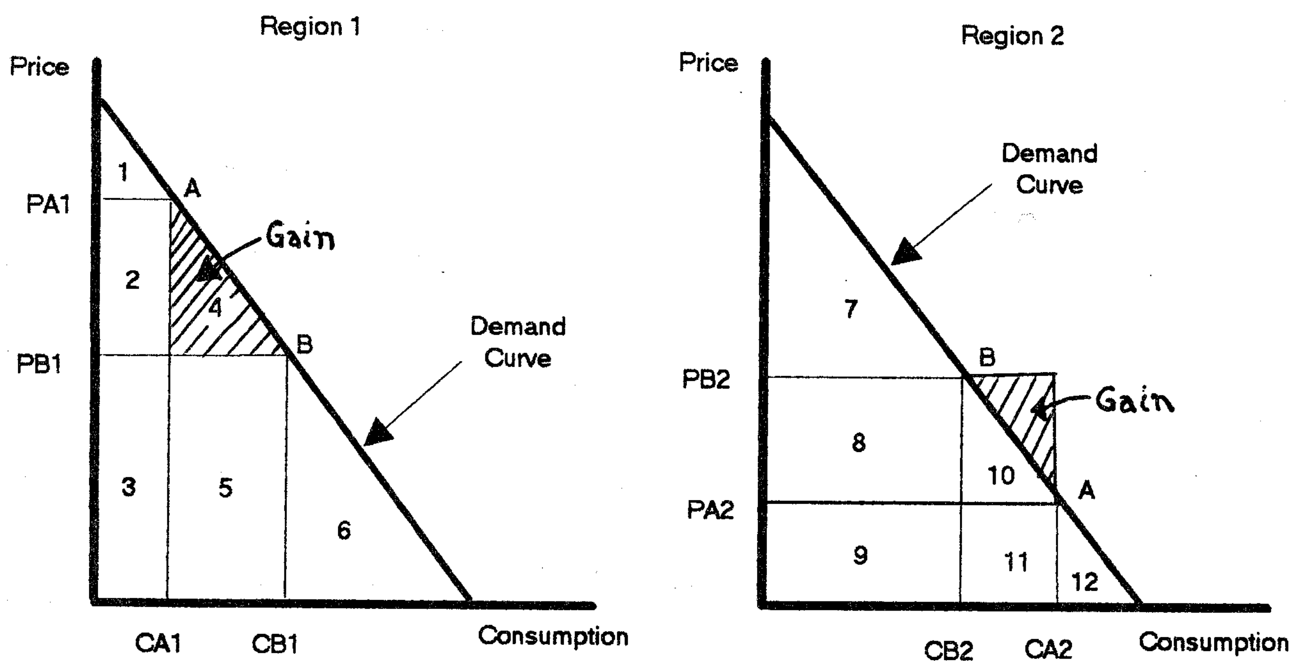

Figure 3.2 Welfare maximization and the 'Law of One Price'.

The previous example relates directly to the situation in which a drought in one region creates a shortage of crop and causes the price in that region to increase. Due to arbitrage from welfare maximizing agents, surplus regions will ship crops into the drought region and reduce the level by which that region's price will increase. Since crops are flowing out of the surplus regions into the drought region, the surplus regions' prices will rise and the drought region's price will fall. As explained earlier, flows of crops will occur to point at which prices in all regions will not differ by more than the transportation costs.

3.2 Demand Curve

Each region has its own demand curve for forage which represents the value placed on forage in that region given regional prices. This value is represented by the areas under the curves (the cumulative value of each unit of forage). For given quantities of forage, regional prices can be established by the respective points on the curve.

The demand curve is a value of marginal product curve, and points on this curve refer to points where the price of the marginal unit of forage equals its value of marginal product in production of livestock. Therefore, when agents maximize welfare, they will purchase forage up until the point where the price of the last unit of forage equals the value of its marginal product. If agents do not purchase forage up until this point, welfare will be measured at some point below the demand curve, where the price of a marginal unit is lower than its value of marginal product. In this case the agents would not be maximizing their own welfare. If all agents maximize welfare then total welfare is maximized and regional prices can be determined by the respective points on the regional demand curves.

The shapes of regular demand curves are typically assumed to be continuous and downward sloping. However, when modelling forage this is not an appropriate shape of curve to use. Since forage crops have a lack of close substitutes, and farmers do not readily alter base herd sizes given small fluctuations in forage prices, the shape of demand curve is not necessarily continuous and downward sloping.

Referring to figure 3.3, the shape of a regional demand curve will be perfectly inelastic (vertical) for a given range of reasonable forage prices. Since farmers have specific stock requirements which they must meet, and there are costs to adjusting herd size (i.e., actual physical costs of buying and selling cattle and the uncertainty associated with it), the demand for forage will not fluctuate given low variability in prices around an average price level. The height of this vertical portion of the demand curve will depend on the distribution and scale of the different farms in the region. If the majority of farmers are operating at the margin, then the vertical portion may kink sooner at the top. These types of farmers will be more responsive to increases in hay prices since their scale of operation is not as profitable (flexible) at the margin as other farms. If the scale of farms in the region is widely distributed, then the upper kink may become smooth as the farms at the margin respond to small price fluctuations and the other farms gradually respond to larger price fluctuations.

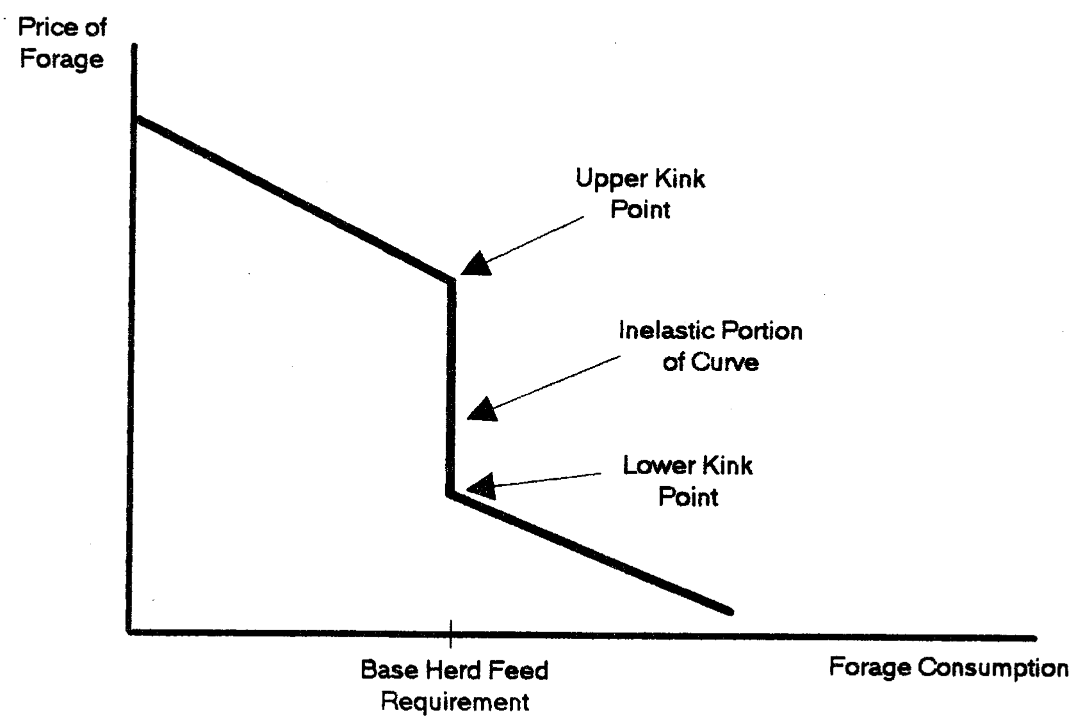

Figure 3.3 Regional Demand Curve for Forage

The regional quantity demanded for forage will likely respond if forage prices become excessively high. Under a wide-spread drought scenario, with below average precipitation across a large area causing decreased forage yields, there may not be adequate stocks in nearby regions to meet the excess demand for forage in that region. As a result, forage prices may become so high that farmers begin to decrease their consumption of forage. This would occur when it is no longer feasible for farmers to maintain and feed their present base herd size given the higher forage prices. The result is a decrease in the demand for forage as farmers seek alternatives (i.e., reduce their base herd size; use more of other grains like barley, if possible; sell off feeder calves earlier than expected; and feed less hay per animal). If the drought is more severe and prices rise to a higher level, the demand for forage will be even less and the demand curve will continue to slope backward to the left.

The demand curve may be kinked at the bottom of the vertical portion since exceedingly low forage prices may entice farmers to increase their base herd size and forage usage. Given farmers present base herd size and feed requirement (as represented by the vertical portion of the demand curve) when forage prices fall low enough (eg., favourable weather patterns across large areas causing surplus forage yields and large drops in forage prices), farmers may choose to invest in livestock. Given the drop in prices, farmers can meet the feed requirements of a base herd larger than the present herd. Although this bottom kink can be debated, it is the upper kink and slope that are of interest for the model (the price responses to drought situations on the upper portion of the demand curve).

A graphical description of a regional demand curve would assume a linear curve intercepting the vertical axis and sloping downward to the right. At the base herd feed requirement level (as described on the horizontal axis), this demand curve becomes downwardly vertical. This represents to the inelastic portion. At the bottom of the inelastic segment, the demand curve slopes down toward the right again.

3.3 Kinked Demand Curve

When using a continuous downward sloping demand curve, it is not necessary to know the section of curve being utilized because one function represents all points on the curve. However, non-continuous (kinked) demand curves require that the section of curve be identified. This is due to a separate linear function used for both the upper and lower slopes of the curve. If optimization occurs to the left of the inelastic portion, then the upper part of the curve is used, and if it occurs to the right of the inelastic portion, then the lower part of the curve is used. The upper portion of the regional demand curves contain separate parameters than the lower curves. The upper portion is linear and downward sloping. In mathematical form it can be described as a vertical intercept (α_u, the subscript 'u' referring to the upper curve), plus a negative slope value (β_u) multiplied by the level of consumption (C, represented on the horizontal axis). The lower portion of the curve is described as the vertical intercept (α_l, the subscript 'l' referring to the lower curve), minus a slope (β_l) multiplied by the level of consumption (C). Note, that the method used in identifying which section of the curve is being referred to is later described in a mathematical description of the model.

3.4 Spatial Aspects

As discussed previously, the model allows regions to allocate forage supplies within and between themselves, and to and from storage, in order to maximize welfare. When dealing with continuous downward sloping demand curves in this type of spatial setting, the results will appear to be uniform (price/quantity relationships are based on continuous curves). However, when incorporating non-continuous (kinked) demand curves like those in this study, the results will differ (price/quantity relationships are based on non-continuous curves).

When regions optimize welfare under a kinked demand curve, the results contain jumps when moving from one section to the other in the demand curve. The vertical intercept (and possibly the slope) are different when referring to the two separate portions of the curve to the left and to the right of the inelastic section. This inconsistency in the demand curve will create different types of solutions in which the 'Law of One Price' holds. In the optimization process, regions will not have the gradually decreasing marginal benefit as consumption increases. Marginal benefit does gradually decrease to a point, as consumption increases; however, an increase in consumption beyond the base herd feed requirement results in a discontinuous fall in the marginal benefit (moving from the upper slope to the lower one).

Regional prices may also exhibit some non-uniform results. When regional consumption levels are to the left of the inelastic portion, pricing is based on the upper slope. And, when consumption is to the right of the inelastic portion, prices are based on the lower slope. But, when regional consumption exactly equals the regional base herd feed requirement, pricing cannot be based on the region's demand curve, as the actual price level is somewhere within the inelastic portion. It is known from the 'Law of One Price' that when regions trade, flows will occur to the point that regional prices will no longer differ by transportation costs. Therefore, the region whose consumption equals its base herd feed requirement can calculate its price from another trading partner's price, plus or minus transportation costs depending on the direction of flow. If the region is importing forage, then its price is equal to the other region's price plus transportation costs, and when exporting forage, its price equals the other's price minus transportation costs.

3.5 Storage

Some dynamic characteristics are incorporated into the model. These characteristics show that farmers at most will look one period ahead into the future, by allocating stocks to storage for use in that period. Given farmers' profit maximizing behaviour, they will arbitrage on forage prices through their use of storage. Assuming that farmers are rational, they will determine whether or not to store production for use in the following period given current forage prices, costs of storing across one period, and the expectation that next period will witness an average price given average production.[18]

Not only are farmers able to send forage stocks into storage, but they also receive stocks from storage. Therefore, when farmers determine how to optimally allocate current production, they include the amount of stocks they are receiving from storage during that period. The model forms no continuous link across time between the amount of forage shipped to storage in one period and the amount of stocks in storage added to production in the following period, since each period is treated as an independent simulation. Given the model has no formal dependency across time, it cannot be considered a truly dynamic model, however, the aforementioned aspects relating to storage do provide some mildly dynamic characteristics to the model.

A steady-state characteristic is necessary when including storage in this model. On the average, the carry-over stocks included in a current period's supply must equal the stocks allocated for use in the following period. Without this characteristic, the results will be biased as current supplies will not equal their true average values. Steady-state for storage is incorporated into the model by running the set of simulations a number of times, until on average stocks in equal stocks out.[19]

Figure 3.4, presents a graphical representation of the demand curve for storage. The expected price next period is represented on the vertical axis with stocks allocated to storage for use next period on the horizontal axis. The curve intercepts the vertical axis and is linearly downward sloping. An interpretation of this curve would be that if no stocks are stored for the period following, an average price can be expected for next period (referring to the point touching the vertical axis). As more and more stocks are allocated for the following period, the expected price next period will fall further and further below the average price. This relates to the curve's negative slope.

---

[18] Note that current forage supplies include carry-over stocks from the previous period.

[19] Carry-over stocks are not included in the first set of runs since they must be generated *a priori*. After the first set of runs are completed, stocks in are included in current supplies for allocation purposes.

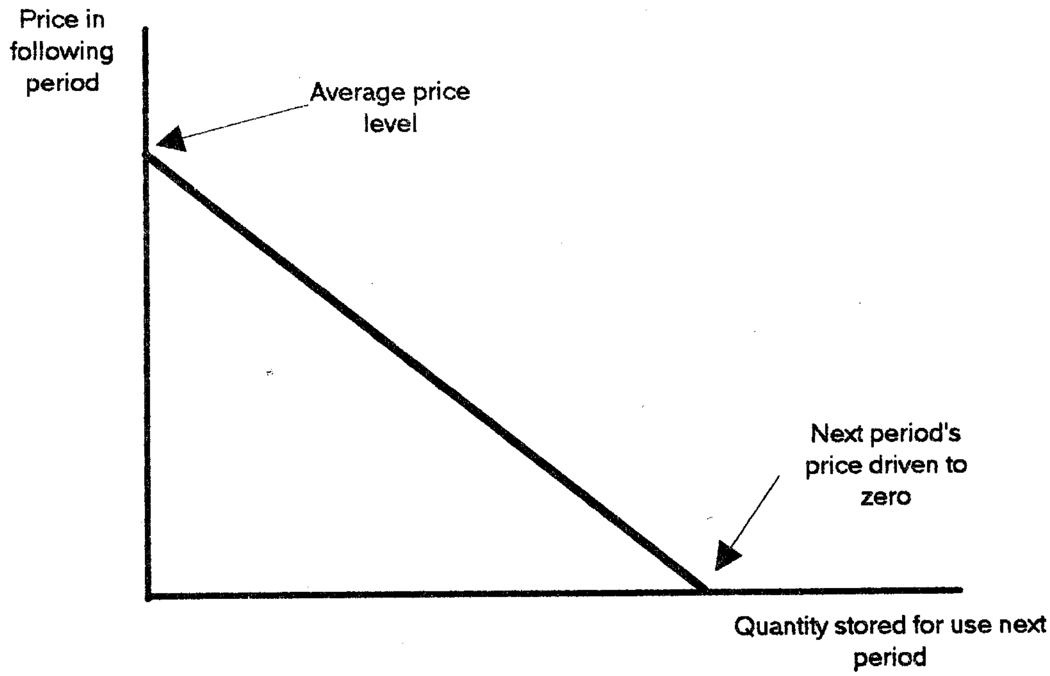

Figure 3.4 Demand Curve for Storage

3.6 Mathematical Model

The model in this study determines the optimal inter/intra-regional allocations of forage crops (for the purposes of consumption) in order to maximize total welfare across regions. This allocation process is based on regional forage supplies and demand and on the assumption that an objective function is being maximized subject to various constraints.[20] This objective function represents total welfare across all regions, where total benefits (i.e., the cumulative area under all regional demand curves) are subtracted from total costs (i.e., the sum of all transportation costs associated with crop flows). The following section presents a description of the model, including the objective function and constraints.

One of the main purposes of the model is to determine optimal allocations

[20] The parameters for the regional demand curves are fixed throughout the optimization, while regional production for each scenario is randomly drawn *ex ante*.

of crops within and across regions. This is done by maximizing an objective function (representing total welfare) subject to various necessary constraints. The first portion of the objective function (representing total benefits) includes mathematical formulas describing the regional demand curves. As described in the previous section, in order to incorporate the regional demand curves into the optimization process, it is necessary to identify to which portions of the demand curves are being referred. Unfortunately, this is not a simple task, as will be shown.

When referring to a point left of the inelastic slope, the area under the curve (welfare benefit) is measured up to the actual level of consumption (C), as represented on the horizontal axis. This area can be calculated in two portions: the triangular area above the actual price ($\alpha_u - \beta_u$*C) and the square area below that price. For a graphical description, see Figure 3.5 below.

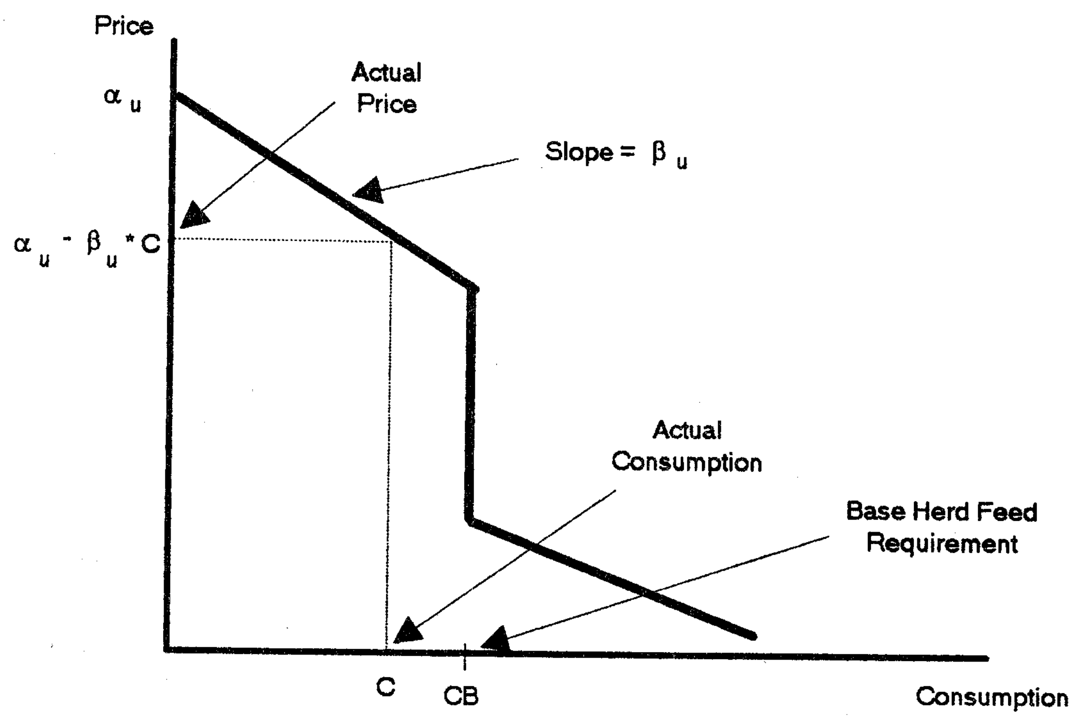

Figure 3.5 Benefit measure under upper slope of demand curve

The following formula represents that area (benefit) under the curve:

$$(3.7.1) \qquad 0.5*[(\alpha_u-(\alpha_u-\beta_u*C))]*C+(\alpha_u-\beta_u*C)*C.$$

This is equal to one-half the difference between the intercept price and actual price, multiplied by the consumption level, plus the actual price multiplied by the consumption level.

When referring to a point right of the inelastic slope, the area (benefit) is also measured up to the actual level of consumption. It includes all of the area to the left of the inelastic portion plus the area between the actual consumption and base herd feed requirement level. See Figure 3.6 for a graphical representation.

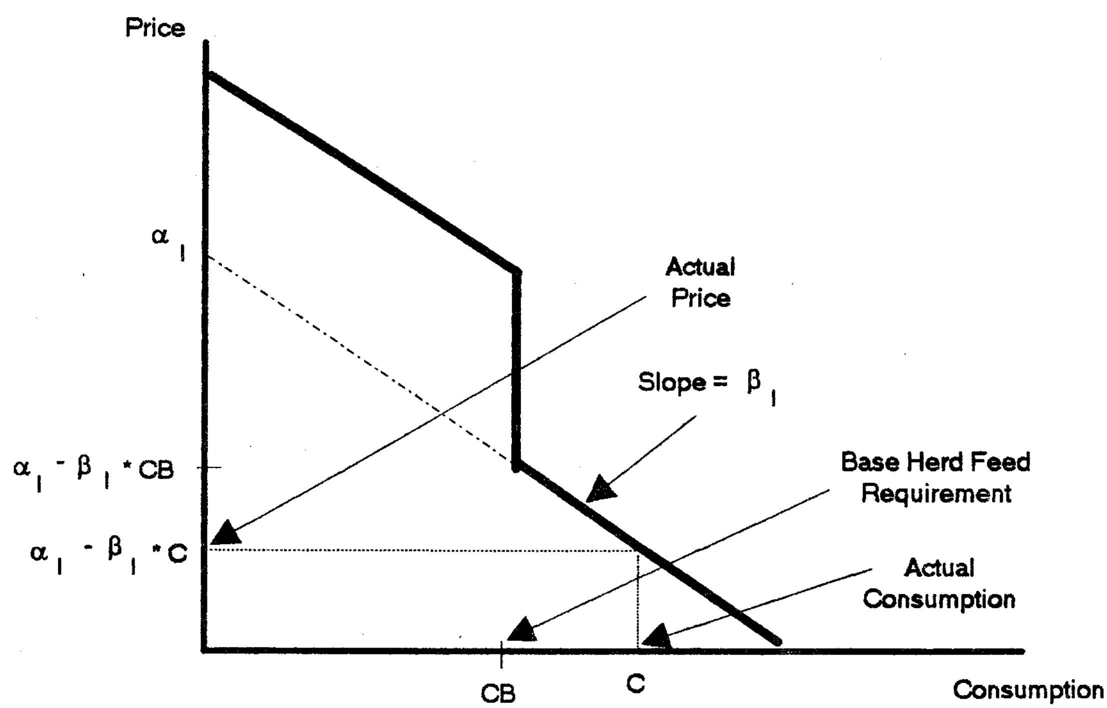

Figure 3.6 Benefit measure for points under lower slope of demand curve

The area left of the inelastic portion is represented by the previous formula, with the actual consumption level (C) replaced by the base herd feed requirement level (CB). The area under the lower slope of the curve, between the inelastic portion and the actual consumption level, is represented by the formula:

$$(3.7.2) \qquad 0.5*[(\alpha_1-\beta_1*CB)-(\alpha_1-\beta_1*C)]*(C-CB)+(\alpha_1-\beta_1*C)*(C-CB).$$

This is equal to one-half the difference between the lower kink price and the actual price, multiplied by the level at which consumption exceeds the base requirement, plus the actual price multiplied by that quantity difference. Therefore, the total area left of the inelastic portion plus the area included to the right is represented by the formula:

$$(3.7.3) \qquad \alpha_u*CB-0.5*\beta_u*CB^2+(C-CB)*[\alpha_1-\beta_1*((C+CB)/2)].$$

Given the demand curve has two distinct sections ((i.e., the portion left and the portion right of the inelastic slope), it is necessary to identify in which section actual consumption falls when welfare is being optimized. Using the two previously described formulas for measuring benefit under the upper and the lower slopes of the curve, identifier parameters can be used for identifying where actual consumption falls. Attached to the previously defined formulas in the objective function are two identifier parameters 'IA' and 'IB'. 'IA' is multiplied by the formula relating to area measurement for points left of the inelastic section, and 'IB' is multiplied by the formula relating to area measurement for points to the right of the inelastic section.

The two identifier parameters are unique in that they only represent a value of one or zero, with neither having the same value. In other words, if 'IA' equals one, then 'IB' equals zero, and only the formula relating to a consumption level less than the base herd feed requirement level is represented in the objective function. If 'IB' equals one, then 'IA' equals zero, and only the formula for a point to the right is represented. These identifiers are successfully able to identify whether consumption falls below or above the base herd level by a series of constraints. Included in these constraints are that the identifiers must be non-negative, not exceed a value of one, and sum to one. Two further constraints are that 'IA' must be less than or equal to zero and 'IB' must be greater than or equal to zero when consumption exceeds the base level. The reverse is true for consumption falling below the base level.

These two formulas (measuring area under the demand curves) and the associated identifier parameters 'IA' and 'IB' are included under the benefits portion of the objective function and are summed over all regions. This represents the total benefits across all regions. The following formula represents this portion of the objective function, in summation notation:

$$(3.7.4) \qquad \Sigma^{n}[IA_n*(\alpha_{un}*C_n-0.5*\beta_{un}*C_n^2)+IB_n*(\alpha_{un}*CB_n-0.5*\beta_{un}*CB_n^2+(C_n-CB_n)*(\alpha_{1n}-\beta_{1n}*(C_n+CB_n)/2))].$$

Note, that the subscript 'u' refers to the demand curve left of the inelastic portion, 'l' refers to the curve right of that portion, and 'n' refers to the region.

The second part of the objective function relates to the costs incurred through the allocation (transportation) of forage crops, within and between regions. These costs are equal to the per unit transportation costs among regions (represented as a matrix) multiplied by the quantity levels transported (also in matrix notation). When these two matrices are multiplied, they represent the transportation costs matrix shown in the objective function, where the costs are represented by all of the trace elements.

The transportation costs matrix is calculated by multiplying the square matrix of per unit transportation costs (A) by the transpose of a square matrix of transportation quantity levels (T'). The per unit transportation cost matrix is represented by each region for both the rows and the columns. This allows for all possible transportation combinations to occur, including allocations within regions. The transpose of the quantity transported matrix is used since its combination with the per unit cost matrix yields a transportation cost matrix in which the elements correctly match up. In other words, the correct per unit costs are attached to the their respective quantity levels, thereby allowing for the costs to be represented as the trace elements in the transportation costs matrix. In matrix notation, this transportation costs matrix can be represented by the following formula:

$$(3.7.5) \qquad \text{trace}(A*T'),$$

where both the per unit cost matrix and quantity level matrix are of dimensions 'n' by 'n'.

Given the previously shown equations, the complete form of the objective function can be stated in the following formula:

$$(3.7.6) \qquad W = \Sigma^n[IA_n*(\alpha_{un}*C_n-0.5*\beta_{un}*C_n^2)+IB_n*(\alpha_{un}*CB_n-0.5*\beta_{un}*CB_n^2+(C_n-CB_n)*(\alpha_{1n}-\beta_{1n}*(C_n+CB_n)/2))] - \text{trace}(A*T').$$

This function represents total welfare across all regions, where the parameter W refers to welfare. It is separated into the sum of all benefits minus the sum of all costs. In the optimization process, this function (total welfare) is maximized subject to a number of constraints, which will be described below.

There are ten constraints in the optimization process. These include a constraint that all of the choice parameters (regional consumption, transportation levels, and the two identifier parameters) be non-negative. This is represented as follows:

$$(1) \quad C_n,\ T_n,\ IA_n,\ IB_n \geq 0,$$

where consumption (C), transportation levels (T), and the identifier parameters are included for all regions.

The second and third constraints are necessary for the allocation process. These include the constraint that a region cannot consume more forage than it possesses and gets shipped in from elsewhere. The following represents this constraint:

(2) $C_n \leq T_{n1}+T_{n2}+\ldots+T_{nn}$,

where the first subscript refers to the destination region for the crop being transported and the second refers to the source of that crop.

The third constraint relates to regions not being able to allocate more forage than they possess (including crops shipped in from elsewhere). This is shown by following formula:

(3) $QP_n \geq T_{1n}+T_{2n}+\ldots+T_{nn}$,

where QP refers to a specific regions supply level (production and carry-over stocks).

The fourth and fifth constraints relate to shipments to and from storage. These are shown by the formulas:

(4) $T_{sn} \geq 0$ and

(5) $T_{ns} = 0$.

The former states that only non-negative quantities can be allocated to storage, while the latter states that quantities cannot be allocated from storage. This allocation process is included separately in the program, as described previously.

The remaining constraints allow for the identifier parameters to function. As explained earlier, two of the constraints ensure that the identifier 'IA' takes on a value of one or zero when consumption falls below the base level, and 'IB' takes on a value of one or zero when consumption exceeds the base level.

These constraints are represented as following:

(6) $IA_n*(C_n-CB_n) \leq 0$, and

(7) $IB_n*(C_n-CB_n) \geq 0$.

Another constraint ensures that the identifiers sum to one. This is shown by:

(8) $IA_n+IB_n = 1$.

The last two constraints force the identifiers to assume values not greater than one. They are represented by:

(9) $IA_n \leq 1$ and

(10) $IB_n \leq 1$.

For further reference, a summary list of the parameters used in the program is shown below:

W = total welfare

α_u = vertical intercept for upper slope of demand curve

α_l = vertical intercept for lower slope of demand curve

β_u = slope of upper portion of demand curve

β_l = slope of lower portion of demand curve

CB = regional base herd feed requirement (horizontal value of inelastic portion of demand curve)

C = actual regional consumption of forage

QP = regional supply of forage (includes production and carry-over stocks)

A = vector of transportation costs within and between regions

T = transportation quantity levels of forage

IA = identifier parameter for a point to the left of the inelastic slope (assumes a value of 1 if upper slope of demand curve is used and 0 if not)

IB = identifier parameter for a point to the right of the inelastic slope (assumes a value of 1 if lower slope of demand curve is used and 0 if not)

3.7 Simulation Process

As described previously, the model used in this study is a simulation model and not an econometric one. It randomly generates data by which the model performs its numerous simulations. A description of this simulation procedure will be presented below.

The model begins by generating random correlated production levels for the regions. To get these values, the model generates production levels for the regions, assuming no correlation between regional production. To do this, a vector of average production levels of all the regions is added to a vector of independent, multivariate normal draws (with means of zero and variances of 1). This allows random independent quantities to be generated, with the mean values for each region's production taken into account.

The correlations between the random production draws is accounted for by multiplying this generated vector of regional production levels by a Cholesky decomposed matrix.[21] The Cholesky decomposed matrix is a nonsingular triangular matrix that has the property that when multiplied by a vector of independent random normal draws will create a vector of correlated random draws. The Cholesky matrix is created by decomposing the variance/covariance matrix of regional production. In simplicity, the variance/ covariance matrix is created from a data series of annual regional production levels. From this matrix, a matrix of characteristic vectors and diagnol matrix of characteristic roots can be found. The two later matrices are multiplied, and together, have identical properties to the nonsingular triangular matrix described above.

Given these random draws for regional production, the model finds the optimal allocations for forage. This is done by maximizing the previous

---

[21] See Judge (1988), p.494-496.

objective function subject to the constraints. Note that the 'Law of One Price' is assumed throughout this optimization procedure, since welfare maximization ensures that this law holds. The determined regional consumption levels are then related to the regional demand curves to obtain the regional prices.

This procedure is performed over numerous simulations in order to obtain a series of regional prices and their associated quantities. Regression analysis (OLS) is then used to draw relationships between the prices and quantities. These results can further be used for the purposes of estimating regional prices given actual quantities.

This pricing model has application for use in a Wide-Spread Drought insurance scheme for British Columbia forage producers. As noted in the introduction, British Columbia forage producers face price risk associated with wide-spread drought. It is for this reason that a Wide-Spread Drought insurance scheme has been proposed, and subsequently a pricing mechanism needed. The following chapter will present an application of this model to the pricing of forage in British Columbia for the purposes of a Wide-Spread Drought insurance scheme.

Chapter 4 : Model specification for British Columbia

4.0 Introduction

The following chapter presents an application of the pricing model to British Columbia forage. Included are a breakdown of the regions, a description of the data used, and an explanation of the computer algorithms to perform the simulations. The following two chapters present some of the results obtained and the conclusions regarding the model's application.

4.1 Application

The province of British Columbia was broken down into five separate regions: Peace River Region; Central Interior Region; Cariboo-Chilcotin Region; Thompson-Okanagan Region; and Kootenay Region, with each region considered both a producer and consumer of forage. This regional breakdown was determined by British Columbia Ministry of Agriculture, Fisheries and Foods (the study's primary funding agents) and based on the regional boundaries defined in Statistics Canada #95-393D (1991). Figure 4.1 shows a rough approximation of the boundaries for British Columbia.

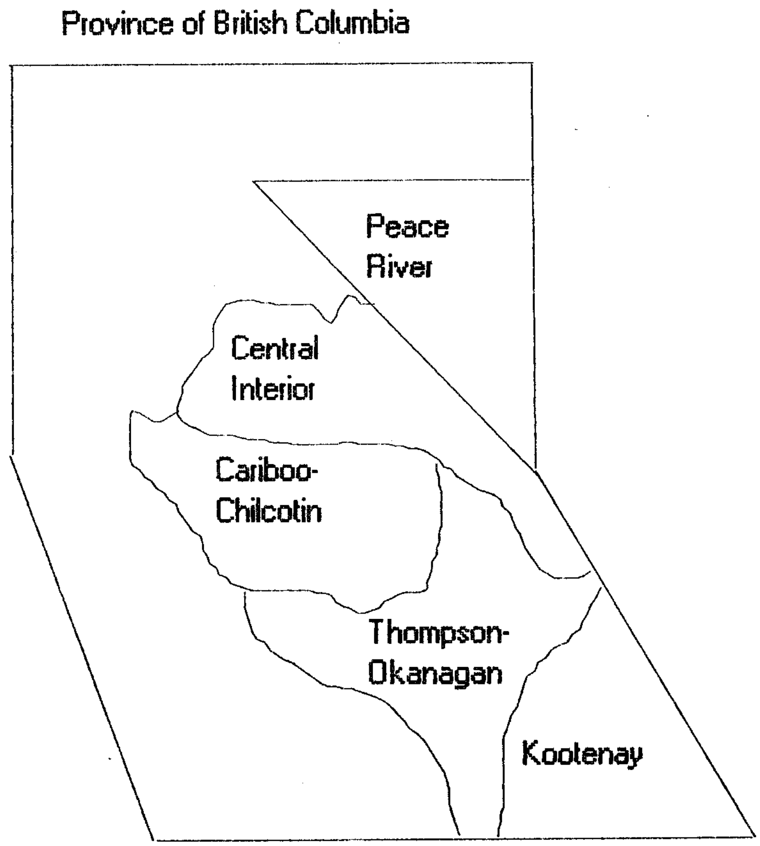

Figure 4.1 Regional breakdown for British Columbia

The Peace River Region includes the Peace River Regional District. The Central Interior Region includes both the Bulkley-Nechako Regional District and Fraser-Fort George Regional District. The Cariboo-Chilcotin Region includes the Cariboo Regional District. The Thompson-Okanagan Region includes the Squamish-Lillooet Regional District, Thompson-Nicola Regional District, Okanagan-Similkameen Regional District, Central and North Okanagan Regional District, Columbia-Shuswap Regional District, and Kootenay Boundary (Subd. B). The Kootenay Region includes the Central and East Kootenay Regional District, and Kootenay Boundary (Subd. A).[22]

[22] The model did not include any regions outside of British Columbia. Some forage is transported between Alberta and the Peace River and Kootenay Region; however, that link was excluded since British Columbia is a net exporter of forage (droughts in Alberta not having as much impact on forage prices in B.C. than if B.C. was a net importer) and the inclusion of Alberta would considerably complicate the problem (more regions to include, data to collect, and variables for which to solve).

Although the model in this study does generate its own data for simulation purposes, there are some actual data requirements that must be met in order for the model to become operational. These requirements refer to data on demand for forage, production of forage, transportation costs within and between regions, and the costs of storage. The data and its sources are listed below.

Information relating to demand for forage was needed for the study. It was assumed that regional consumption (demand) of forage is determined by the regional base herd feed requirements. These feed requirements are based in turn on both the regional livestock numbers and feed requirements per animal (for which data were collected).

The livestock numbers included the provinces main forage consuming animals: cattle, horses, and sheep (lambs). The regional numbers are found in Table 1 of the Appendix and were obtained from Statistics Canada #95-393D (1991). Feed requirements for each category of animal are found in Table 2 and were obtained from: Keay (1991); Agriculture Canada (1986); Ross (1989); and National Research Council (1989). This table also includes the regional base herd feed requirement values used in the study.

Further information on demand for forage was needed in order to parameterize the regional demand curves. Average regional long-term price estimates for forage were obtained from various forage experts around the province, see Aumack et al. (1994). These same individuals provided insight into the responses of average farmers to fluctuations in forage prices. It was from these rough estimates and information that the model's parameters were calibrated.[23]

Production data was also needed. This related to information describing average regional production and the variance/covariance of production. Average regional production was calculated from data characterising regional yields per

---

[23] No series data on forage consumption was available to obtain graphical estimates of the demand curves.

acre and seeded acreage. The yield data is shown in Table 3 and was obtained from: Statistics Canada #22-201 Annual Statistics and Grain Trade of Canada (1990), Statistics Canada #22-002 (1991), British Columbia Ministry of Agriculture, Fisheries and Foods (1994). Seeded acreage data is found in Table 4 and was acquired from Statistics Canada #95-393D (1991). Also, included is the average regional production values used in the study.

There was no series data available on regional yields, and therefore no direct means for calculating production variability. As a result, some other means was needed for calculating the variance/covariance in regional yields. An assumption that the variance/covariance in regional precipitation affected regional yields was made. This assumption was relatively realistic since fluctuations in precipitation are the main impetus behind variability in yields. Therefore, given series of monthly regional precipitation levels, coefficients of variation and correlation coefficients for regional precipitation were calculated. Assuming a relationship of 0.3 between the variation in precipitation and variation in yield[24], a variance/covariance matrix was established for the regions. The variance/covariance matrix was then transformed via the Cholesky matrix decomposition method into a nonsingular triangular matrix ρ, which was then used for randomly drawing regional production.[25] The coefficient of variation/ correlation coefficient matrix for the rainfall data is found in Table 5 and the triangular decomposed matrix is in Table 6.

Transportation costs were also a necessary data requirement. A matrix of transportation costs per ton of forage within and between regions are found in Table 7. These figures were calculated using transport cost quotes from various trucking companies throughout British Columbia, see McConughy et al. (1994). The average distances between regional centres is found in Ministry of Tourism (1987). The regional centres were chosen by the Crop Insurance Branch at British

[24] No studies or data were available to make this exact relationship. The value of 0.3 was used since it appeared reasonable in comparison to other values, and yielded acceptable results.

[25] The Cholesky matrix decomposition method is found in Judge (1988) and White (1993).

Columbia Ministry of Agriculture, Fisheries and Foods, and represent the most intense areas in each region for production and consumption of forage. The regional centres are as follows: Fort St. John (Peace River Region); Vanderhoof (Central Interior Region); Williams Lake (Cariboo-Chilcotin Region); Kamloops (Thompson-Okanagan Region); and Cranbrook (Kootenay Region).

Storage and carry-over stocks are also included in the model, therefore, information on actual physical costs associated with storage are a necessary data requirement. Storage costs relate to the value of stored forage lost due to spoilage. The costs included in the study are shown in Table 8 and represent a percentage of the stored forage that is lost from spoilage, multiplied by the value of that forage. Information on the costs of storage were obtained from British Columbia Ministry (1994) and Soder (1976).

4.2 Computer Algorithm

In this section, the computer algorithms in the study are explained. The main portion of the computer calculations were made using the GAMS computer package, see Appendix 3. The program starts by defining three separate files in which the output is sent. Three sets are then defined, labelling the five regions, the number of iterations or simulations to complete, and a set used in a loop to solve for price. The parameters of the model describing the regional demand curves, base herd feed requirement, and mean regional production are entered using a series of 'parameter' commands.

The parameter 'IND' and the Cholesky decomposed matrix[26] used in randomly drawing production levels, are entered. Two separate set of parameter definitions help to generate the random production levels. The first definition assumes that the random production levels for the regions are equal to the Cholesky matrix multiplied by a vector of non-correlated normal random draws (with mean 0 and variance 1), added to the mean production levels. The second assumes that positive production levels are represented.

---

[26] Calculated previously using the SHAZAM computer package, see White (19(1993). Note: this algorithm is also found in Appendix 3.

The parameters S1 through S5 are used in storing carry-over stocks to be included in the next 150 simulations.[27] These simulations are repeated six times in order for current supply levels to converge to their true values (steady-state to be imposed on carry-over stocks). A matrix of the transportation costs within and between regions is then entered. The objective function and constraints described previously are entered, followed by a definition of the model 'Version1' and the solve command. The allocations to storage (for use in the following period) are saved in the parameter S1 and used in the second set of 150 simulations. This is repeated a number of times for the convergence described previously.

The end of the program contains a set of 'if-then' statements to solve for prices. If the optimal consumption level for a region does not equal the base herd requirement, then price is based on the demand curve. If consumption does equal the base requirement (i.e., it is on the inelastic portion of the demand curve), then price is based on another trading region's price plus or minus transportation costs, depending on the direction of flow. The final set of 'put' statements send the regional supply levels and their associated prices to the output files.

Given the generated price and quantity series, regression analysis is used to define the parameters for estimating forage prices from regional supplies of forage. Prices are estimated for each region under each of the 150 simulations by substituting the regional quantities back into the regression equations. These estimated prices can then be used in determining levels of payments under the proposed Wide-Spread Drought insurance scheme.

Since the slopes of the upper portion of the demand curves are not known with certainty and play an integral role in the cost of the program[28], three

---

[27] Carry-over stocks are not included in the first set of 150 simulations, but are included thereafter.

[28] The steepness of the slope of the upper portion relates changes in consumption to changes in price and thus affects the estimated price levels and the cost of the program (i.e., an increase

different slopes are used to see the effect on prices. It is assumed that with a steeper upper slope, the level of estimated prices will increase, thus increasing the frequency and level of payments.

---

in the level of estimated prices will increase the frequency and level of payments under the insurance scheme).

Chapter 5 : Results

The first set of results were obtained with all of the upper slopes of the demand curves set to the same level. This was called the 'Base Case Scenario'. Since a means of determining the actual slopes of the regional demand curves was unavailable, a common slope was used representing an average of all upper slopes with an elasticity of negative one at the upper kink point.[29] Note that the Kootenay Region was given a steeper slope than the common one since its upper kink point (at the base herd feed requirement level) was much closer to the vertical intercept. In other words, it was necessary to give the Kootenay Region a greater slope, since the vertical intercepts to the upper slopes of the demand curves should not necessarily exceed transportation costs between regions.

Before the main results are presented, a typical year (simulation run) will be described to show the flows that can occur in the allocation process. This was done using the 'Base Case Scenario' parameters. In this typical year, the randomly drawn production levels for the regions are as follows: 143,857 tons for the Peace River Region; 262,373 for the Central Interior Region; 104,265 for the Cariboo-Chilcotin Region; 334,676 for the Thompson-Okanagan Region; and, 146,815 for the Kootenay Region. Note that the Central Interior, Cariboo-Chilcotin, and Kootenay regions' production levels include quantities drawn from storage; these values equal 68,194, 15530, and 61,232 tons respectively. Given the regional base herd sizes, the regional base herd feed requirements are 216,675 tons for the Peace River, 162,368 for the Central Interior, 208,734 for the Cariboo-Chilcotin, 249,131 for the Thompson-Okanagan, and 63,461 for the Kootenay regions. As a result, both the Peace River and Cariboo-Chilcotin regions did not produce enough forage to meet their requirements.

With regions able to transport production within and between themselves,

---

[29] There was no concrete reason for deciding that an elasticity of negative one at the upper kink points was valid other than a general acceptance for assuming this by the Crop Insurance Branch at British Columbia Ministry of Agriculture, Fisheries and Foods, and after consultation and debates with hay suppliers and consumers in B.C.

an equilibrium solution will occur when the Central Interior Region ships 72,818 tons of forage to the Peace River Region and 27,187 tons to the Cariboo-Chilcotin Region; the Thompson-Okanagan Region ships 77,282 tons to the Cariboo-Chilcotin Region; and, the Kootenay Region ships 83,353 to storage. The result is that all regions are able to exactly satisfy there base herd feed requirement levels, with the Thompson-Okanagan Region consuming 8,261 in excess of its base requirement level. Note that the differences in regional price levels in this equilibrium solution do not exceed the costs of transportation, where the trading regions prices exactly exceed transportation costs; the price in the regions equals 111 dollars per ton (Peace River Region), 69 (Central Interior), 94 (Cariboo-Chilcotin), 72 (Thompson-Okanagan), and 41 (Kootenay). The price in the Kootenay Region exactly exceeds the storage price 62 (expected price next year) by the cost of storage (due to spoilage).

On average, the Peace River and Kootenay regions produce approximately enough forage to satisfy regional requirements. The Peace River Region produces 195,333 tons of forage and requires tons 216,675, and the Kootenay Region produces 77,925 but requires only 63,461. The Cariboo-Chilcotin Region is a significant net importer of forage, producing only 111,893 tons but requiring 208,734. The Central Interior and Thompson-Okanagan regions are both net exporters, with the Central Interior producing 193,839 tons and consuming 162,368, and the Thompson-Okanagan producing 318,272 and consuming 249,131.

As a result of this mismatch in supply and demand for forage, crops will flow between and within regions. Note that storage is excluded from this example as steady-state for storage is assumed. The Peace River Region satisfies its excess demand by importing 21,342 tons of forage from the Central Interior. Cariboo-Chilcotin Region imports 10,129 from the Central Interior and 83,605 from the Thompson-Okanagan. Kootenay Region exports 14,464 to the Thompson-Okanagan. The result is that all regions more or less satisfy their forage requirements.

Using the 'Base Case Scenario' parameters, 150 simulation runs were created. The results can be seen in Figures 5.1 to 5.5. The graphs show a plot of each region's predicted prices[30] on the region's supply levels. Also included are the average predicted price and production levels. Note that a typical simulation for the 'Base Case Scenario' was shown in the previous example.

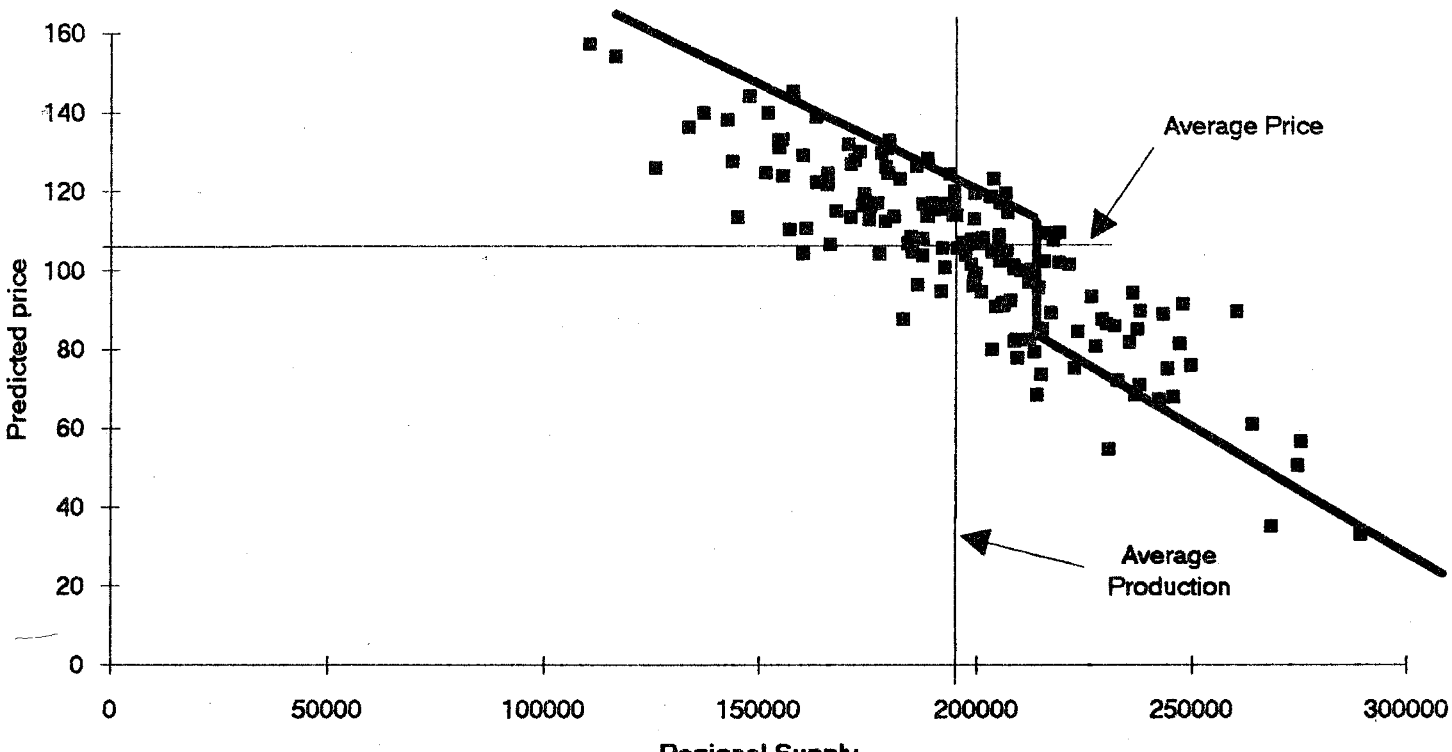

Figure 5.1 Peace River Region (Base Case Scenario)

[30] The predicted price values are similar to the actual prices. A description of the method used for predicting prices will follow later.

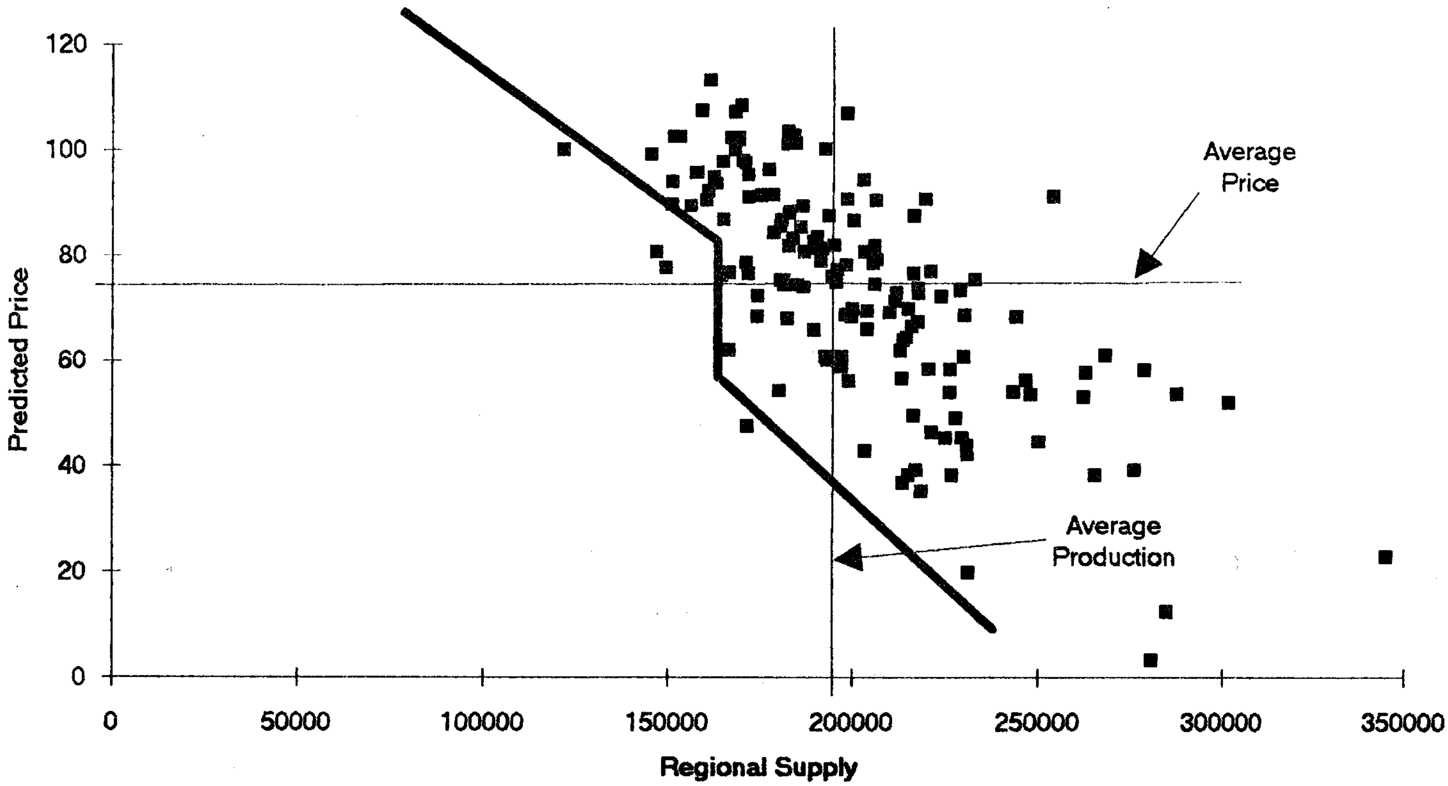

Figure 5.2 Central Interior Region (Base Case Scenario)

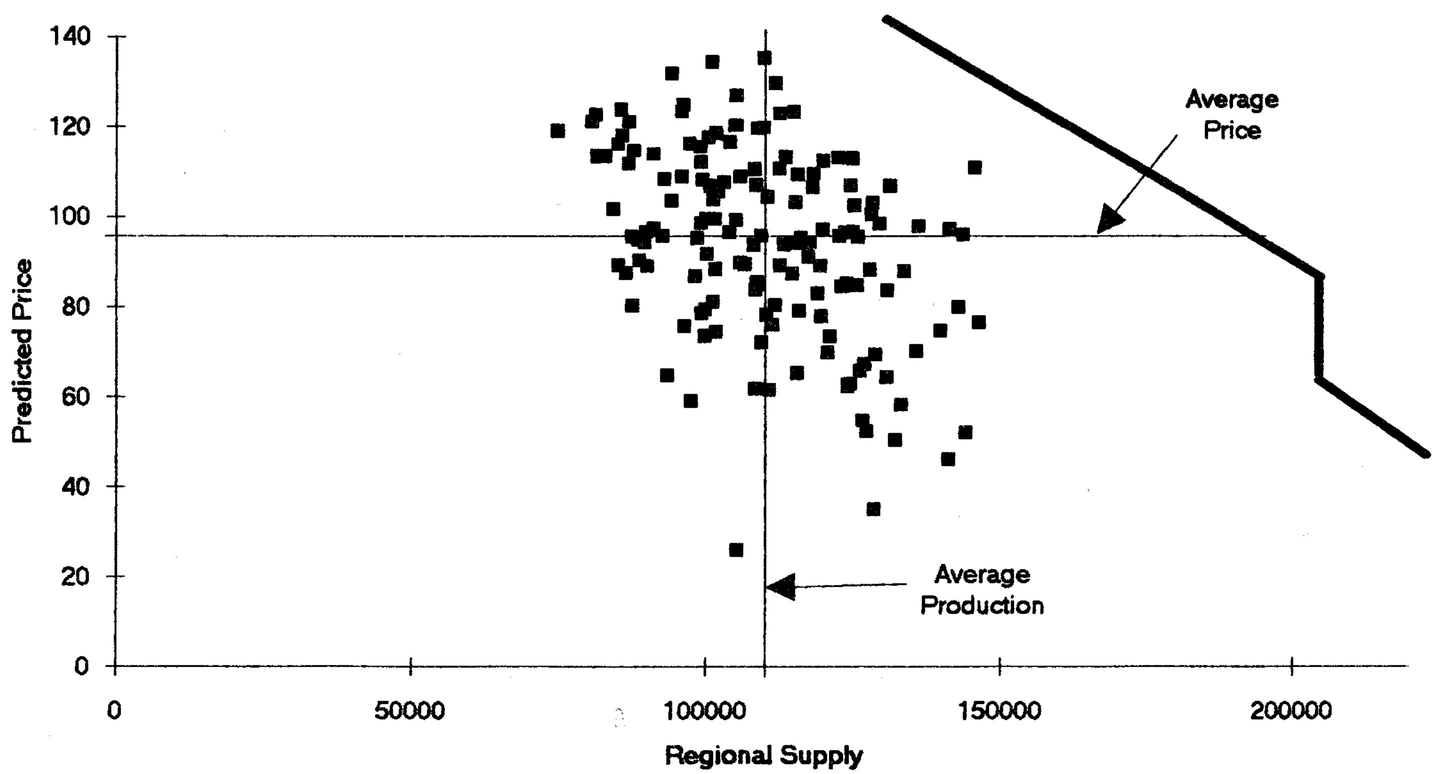

Figure 5.3 Cariboo-Chilcotin Region (Base Case Scenario)

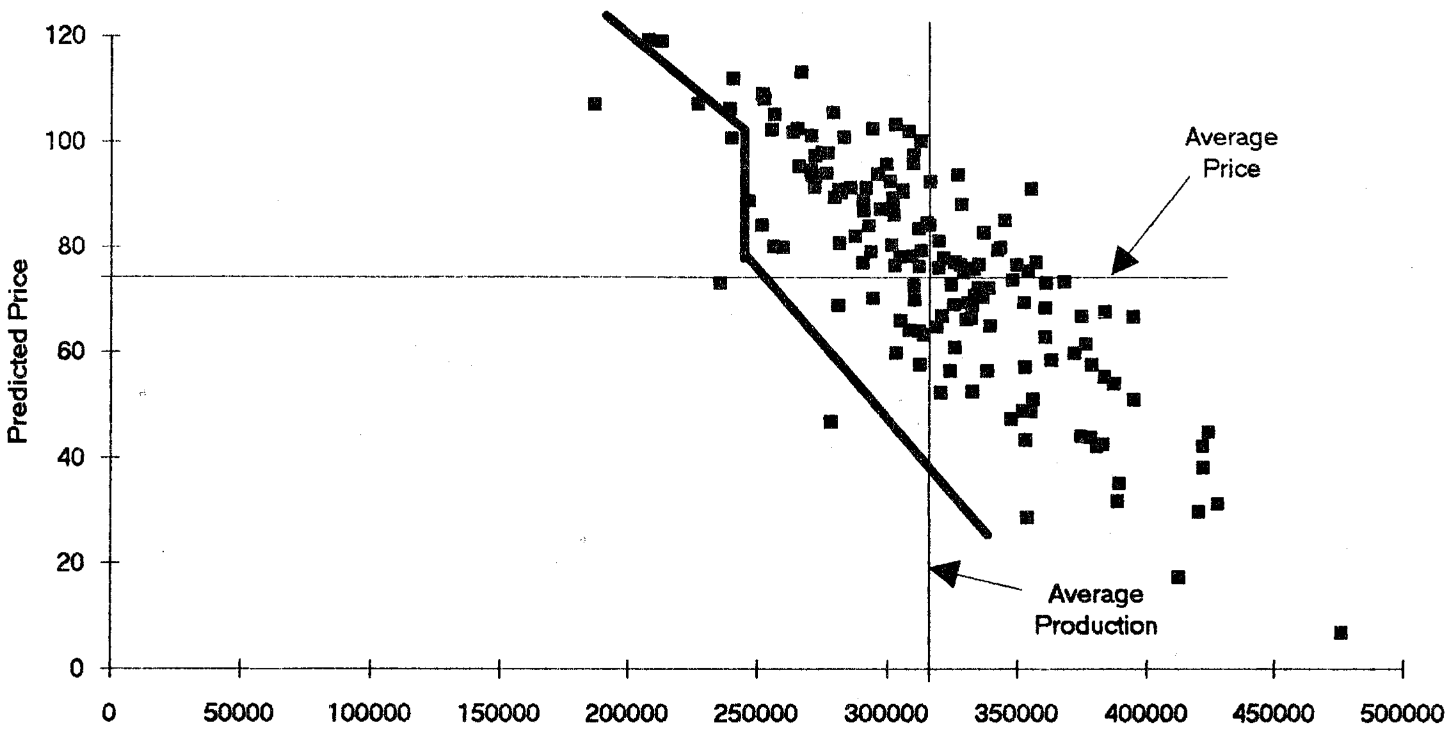

Figure 5.4 Thompson-Okanagan Region (Base Case Scenario)

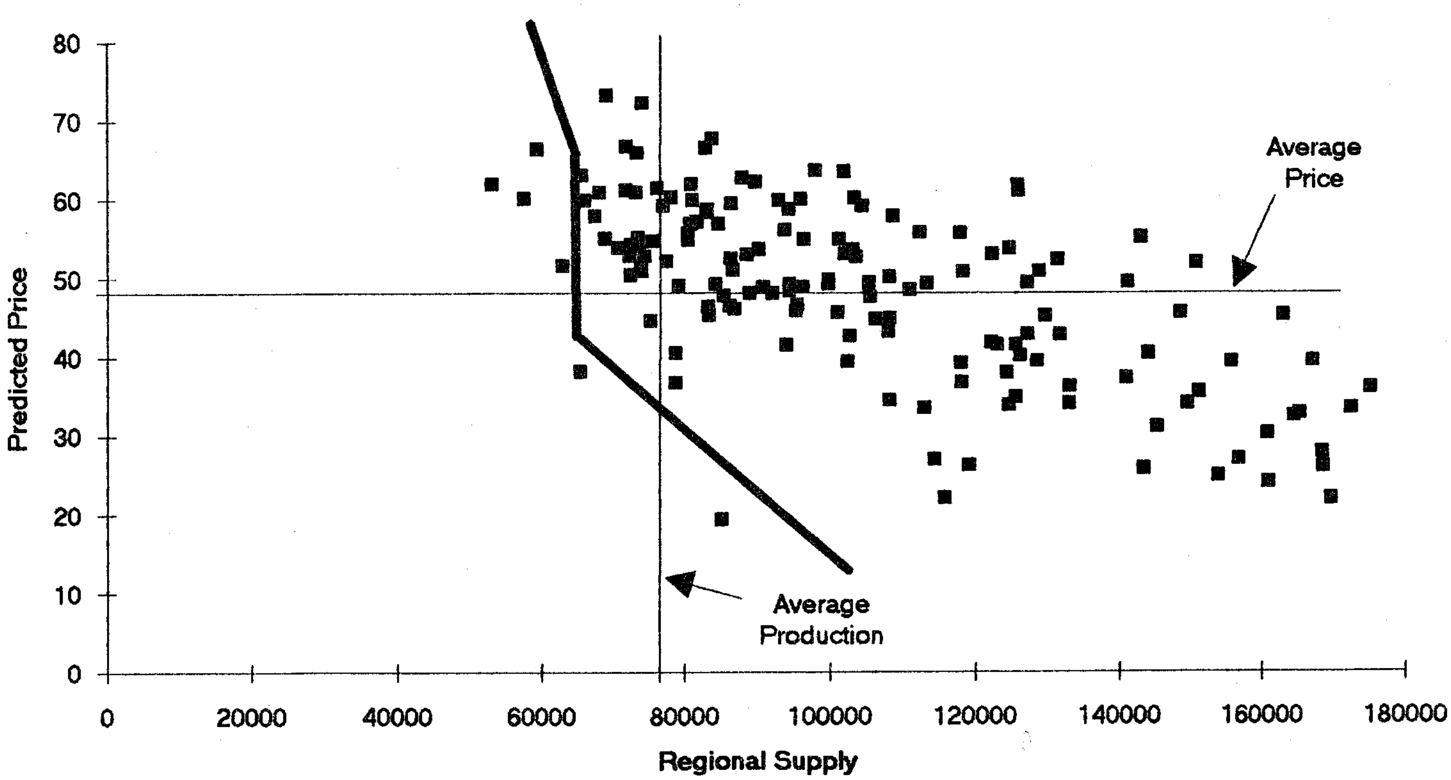

Figure 5.5 Kootenay Region (Base Case Scenario)

From the graphs, predicted prices appear to rise as production levels fall as shown by the downward sloping trends. Since predicted prices are based on all regional production levels, and only one region's production levels are represented in each graph, the observation do not exhibit a perfect trend (i.e., the discrepancy in the trends of the observations can be attributed to the impact of other regions' production levels on the price levels of the region in question).

The predicted prices for the Peace River Region ranged from a low of 35 dollars per ton to a high of 160, given an average of 108. The supply levels for the Peace River Region from the 150 simulations ranged from 110,000 to 290,000 tons. The Central Interior Region had predicted prices ranging from 4 to 118, given an average of 75, with supply ranging from 125,000 to 350,000. The Cariboo-Chilcotin Region had prices ranging from 25 to 135, given an average of 96, with supply ranging from 70,000 to 150,000. The Thompson-Okanagan Region had prices ranging from 8 to 120, given an average of 76, with supply ranging from 180,000 to 475,000. The Kootenay Region had prices vary from 20 to 75, given an average of 48, with supply varying from 55,000 to 175,000.

From the 150 simulation runs, regional prices and their associated quantity levels were obtained. Regression analysis was used to draw a relationship between these prices and quantities. Note that each regional price series was regressed on all of the regional quantities using a linear OLS regression technique. The regression equations are shown below.

The results appear quite reasonable, with R-squared values ranging from 0.59 for predicting price in the Peace River Region to 0.84 for the Cariboo-Chilcotin Region. The t-stat values (in brackets) also appear reasonable, with values greater than two for all but two cases. Given the significance of the t-stats, the Thompson-Okanagan Region appears to be one of the more significant regions affecting prices. This is not surprising since it is a considerably large exporter of forage, on the average. Note: 'P' refers to the region's price, 'QPe' to production in the Peace River Region; 'QCe' production in the Central Interior; 'QCa' production in the Cariboo-Chilcotin; 'QTh' production in the Thompson-Okanagan; and 'QKo' production in the Kootenay.

Peace River :

$$P=322-\underset{(14.6)}{0.0005}*QPe-\underset{(5.6)}{0.0002}*QCe-\underset{(3.0)}{0.0002}*QCa-\underset{(4.9)}{0.0001}*QTh-\underset{(2.4)}{0.00009}*QKo$$
$R^2=0.73$

Central Interior :

$$P=294-\underset{(7.8)}{0.0002}*QPe-\underset{(13.0)}{0.0003}*QCe-\underset{(5.0)}{0.0003}*QCa-\underset{(10.6)}{0.0002}*QTh-\underset{(4.4)}{0.0001}*QKo$$
$R^2=0.81$

Cariboo-Chilcotin :

$$P=309-\underset{(5.2)}{0.0001}*QPe-\underset{(11.2)}{0.0003}*QCe-\underset{(6.4)}{0.0003}*QCa-\underset{(16.0)}{0.0003}*QTh-\underset{(5.7)}{0.0002}*QKo$$
$R^2=0.84$

Thompson-Okanagan :

$$P=291-\underset{(2.5)}{0.0001}*QPe-\underset{(8.7)}{0.0002}*QCe-\underset{(6.1)}{0.0003}*QCa-\underset{(16.4)}{0.0003}*QTh-\underset{(5.2)}{0.0002}*QKo$$
$R^2=0.83$

Kootenay :

$$P=150.4-\underset{(1.3)}{0.00004}*QPe-\underset{(3.5)}{0.00009}*QCe-\underset{(1.9)}{0.0001}*QCa-\underset{(8.4)}{0.0002}*QTh-\underset{(6.5)}{0.0002}*QKo$$
$R^2=0.59$

Since a linear regression technique was used in creating the results above, an interpretation of the parameter values is relatively meaningless as actual value changes are represented. A more useful interpretation could be made if the parameter values represented percentage changes (or elasticities). Therefore, the regressions were re-done using a log-linear form, since the associated parameter values would represent percentage changes.

The following equations show the results of the log-linear regressions used in relating regional prices to all of the regional quantities. These are the equations that the Ministry of Agriculture would use in predicting regional price levels. The following variables are defined as :

%dQPE = the percentage deviation between actual and average production in the Peace River Region

%dQCE = the percentage deviation between actual and average production in the Central Interior Region

%dQCA = the percentage deviation between actual and average production in the Cariboo-Chilcotin Region

%dQTH = the percentage deviation between actual and average production in the Thompson-Okanagan Region

%dQKO = the percentage deviation between actual and average production in the Kootenay Region

Also, let '%dPPE' denote *the percentage deviation between actual and average regional price in the Peace River Region*; similar price deviation variables are defined for the other regions.

Log-Linear regressions for 'Base Case Scenario' :

%dPPE=-1.05%dQPE-0.41%dQCE-0.25%dQCA-0.40%dQTH-0.11%dQKO

%dPCE=-0.71%dQPE-1.12%dQCE-0.46%dQCA-0.93%dQTH-0.22%dQKO

%dPCA=-0.36%dQPE-0.68%dQCE-0.37%dQCA-0.96%dQTH-0.17%dQKO

%dPTH=-0.39%dQPE-0.72%dQCE-0.55%dQCA-1.36%dQTH-0.24%dQKO

%dPKO=-0.22%dQPE-0.41%dQCE-0.24%dQCA-1.00%dQTH-0.40%dQKO

The previous equations can be interpreted as the effect that percentage changes in all regional production has on the percentage change in each regional price. For example, if production was 10 percent below normal in all regions, then price would be 22.2 percent above normal in the Peace River Region. This value is calculated by multiplying -10% by the parameter associated with each region in the equation. Summing these values gives the cumulative effect of all regional production levels being 10 percent below normal on the price in the Peace River Region. Similarly, a 10 percent shortfall in production in all regions would lead to a price rise of 34.4 percent above normal in the Central Interior, 25.4 percent above normal in the Cariboo-Chilcotin, 32.6 percent above normal in the Thompson-Okanagan, and 22.7 percent above normal in the Kootenay regions.

As described at the end of Chapter 4, the slopes of the regional demand curves was not known with certainty. The slopes of the demand curves, especially the upper ones play an integral role in determining the levels of the predicted prices, and this in turn can have an impact on the size and frequency of payouts under the WSD insurance scheme. It is for this reason that sensitivity analysis was performed on the upper slopes of these regional demand curves. This sensitivity analysis was done by increasing and decreasing the upper slopes of the curves in the scenarios labelled 'Steep Slope Scenario' and 'Flat Slope Scenario', and re-doing the log-linear regressions to compare the effects on price with that of the 'Base Case Scenario'.

The same log-linear regression procedure was used in the second scenario. In this case, the upper slopes of the regional demand curves were slightly higher (the vertical intercept on the upper slope was higher by 50 units from the 'Base Case Scenario', for each region). This scenario was labelled as the 'Steep Slope Scenario'. It was assumed that by increasing the slopes, higher price estimates would be generated. See Appendix 2 for details.

A comparison between the results from the 'Base Case Scenario' and the 'Steep Slope Scenario' validates the assumption that increasing the slopes yields larger price estimates (i.e., steeper sloped demand curves are more price responsive). For the 'Steep Slope Scenario', the result of a 10 percent shortfall in production below normal yielded an increase in regional prices above normal by 25.1 for the Peace River, 40.0 for the Central Interior, 24.6 for the Cariboo-Chilcotin, 33.8 for the Thompson-Okanagan, and 29.5 for the Kootenay regions. For all of these cases except for the Cariboo-Chilcotin Region, the change in price was greater than that under the 'Base Case Scenario'. For this region, the 'Steep Slope Scenario' yielded a one percent lower increase in price than the 'Base Case Scenario', which could probably be attributed to an inaccuracy in parameterizing the regional demand curves, causing a slight bias in the allocation process (i.e., slightly more forage allocated to Cariboo-Chilcotin region when slopes are increased resulting in a less responsive price). This is, however, of no great concern as all other regions respond correctly and the violation in the Cariboo-Chilcotin price is negligible in size.

In the third scenario, the upper slopes of the regional demand curves was decreased by 50 units from the 'Base Case Scenario', for each region. It was assumed that this would result in smaller price estimates (i.e., smaller sloped demand curves are less price responsive). See Appendix 2 for details.

A comparison in results between the 'Flat Slope' and 'Base Case' scenarios confirms the hypothesis of the lower slope being less price responsive. For the

'Flat Slope Scenario', a 10 percent shortfall in production below normal for all regions yields an increase in price above normal of 20.3 percent for the Peace River, 29 percent for the Central Interior, 18.3 percent for the Cariboo-Chilcotin, 25.3 percent for the Thompson-Okanagan, and 14.2 for the Kootenay regions. For all of these cases, the increase in price was less than for the 'Base Case Scenario'.

The pricing model appears to respond reasonably well to changes in the slopes of the demand curves. Larger price estimates resulted from an increase in the slope in four of the five regions, confirming that steeper slopes are more responsive; and, lower price estimates for all regions resulted when slopes were lowered, showing that lower slopes are less price responsive. Next, the accuracy of the price estimates was checked.

Using the linear regression results from the 'Base Case Scenario', predicted price estimates were obtained by substituting the regional quantity levels from the 150 simulations back into each regression equation. A graphical display showing the accuracy of the price estimation is found below in Figures 5.6 through 5.10. These graphs represent the predicted price over the actual price for each of the 150 simulations. A value of one represents a perfectly approximated regional price.

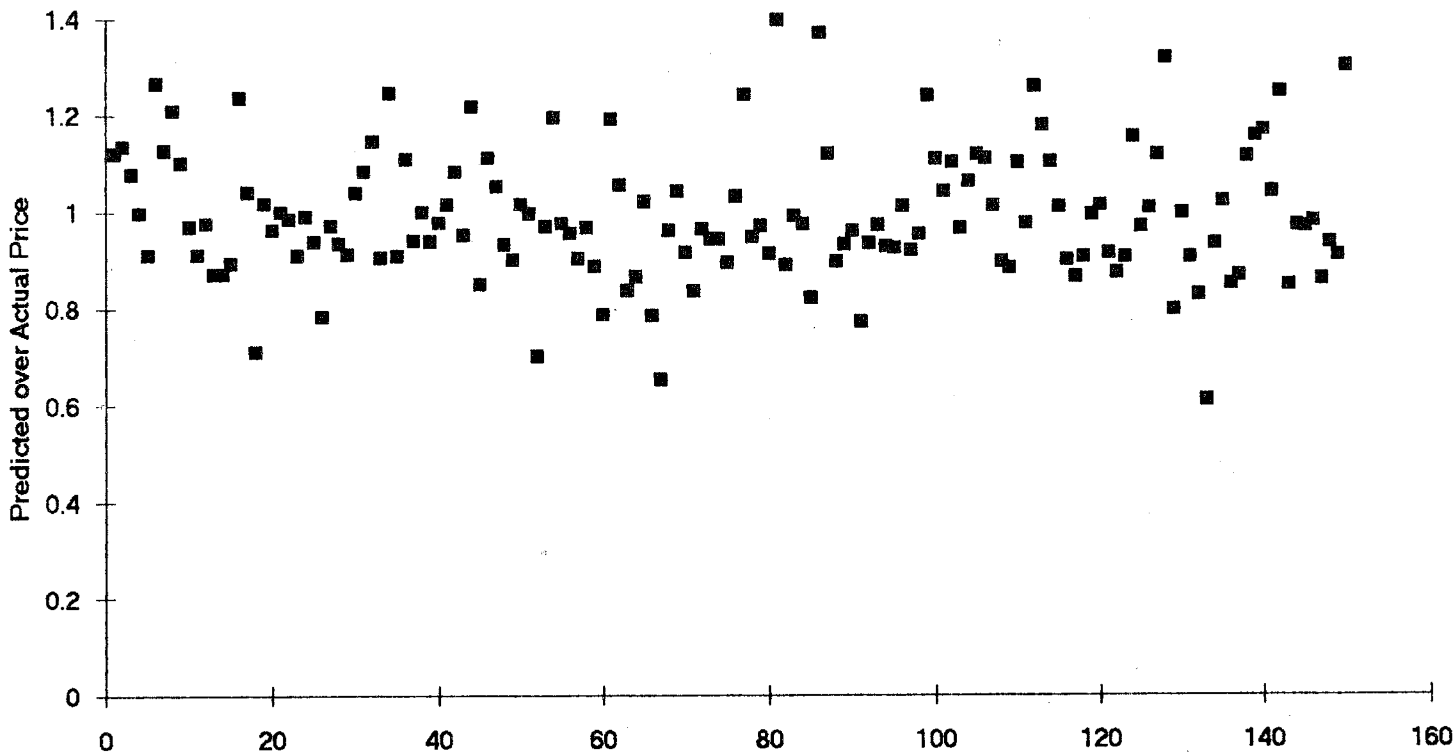

Figure 5.6 Predicted over real price for Peace River Region

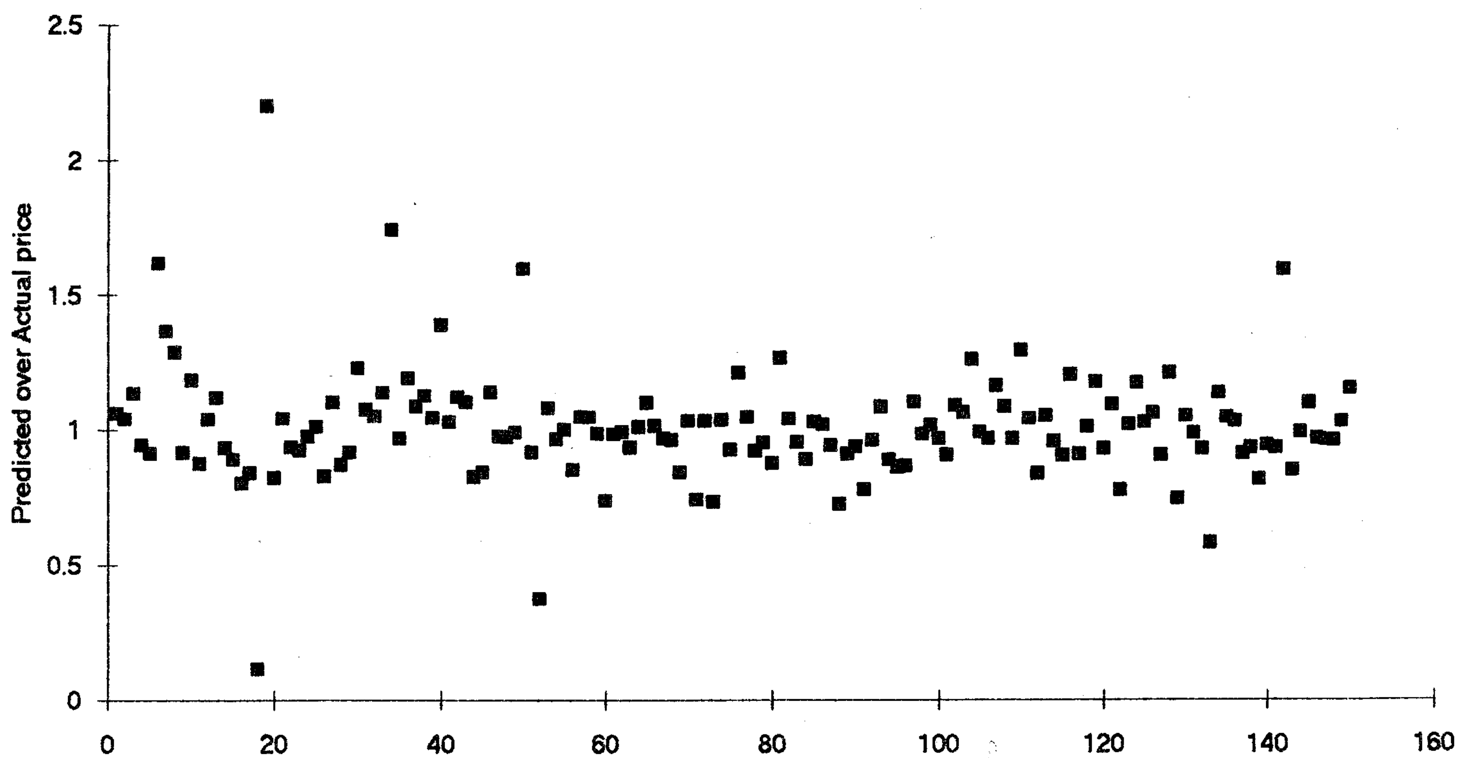

Figure 5.7 Predicted over real price for Central Interior Region

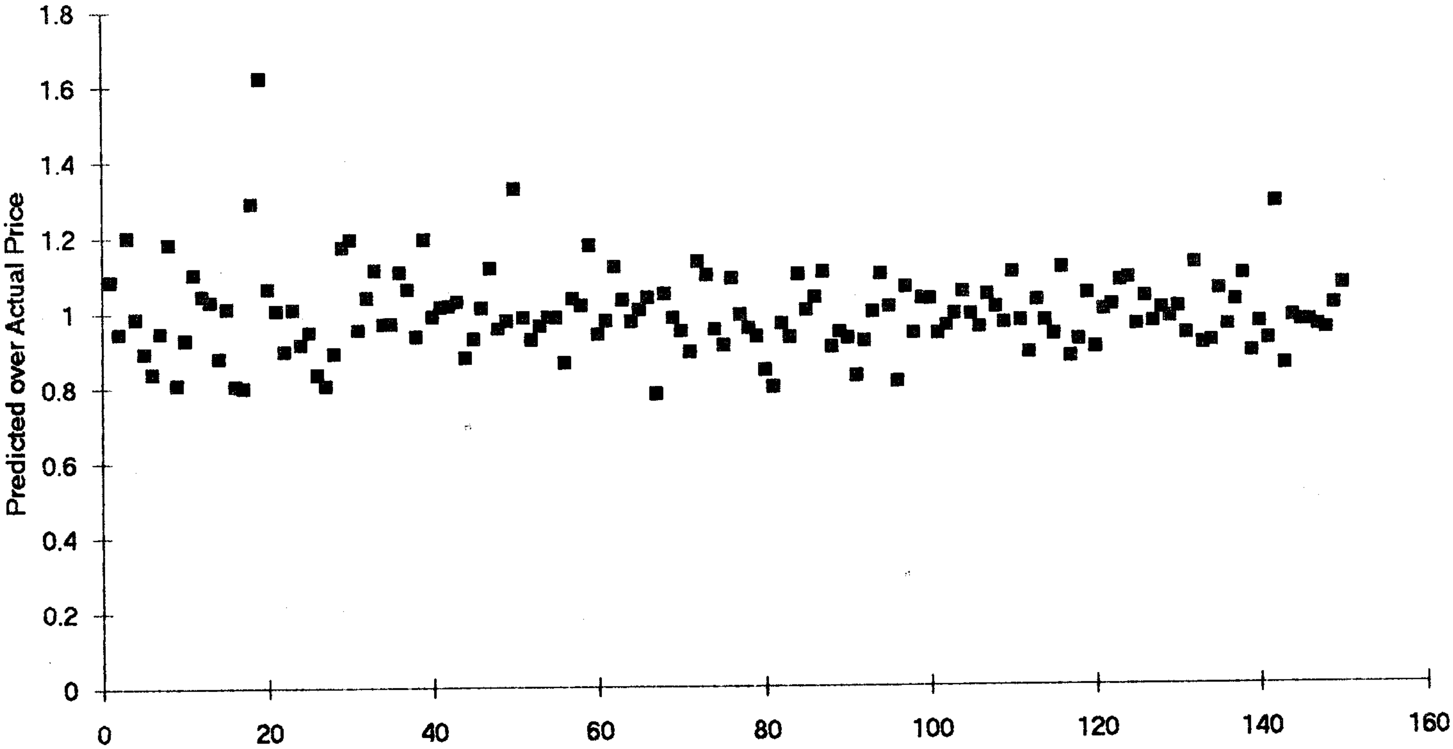

Figure 5.8 Predicted over real price for Cariboo-Chilcotin Region

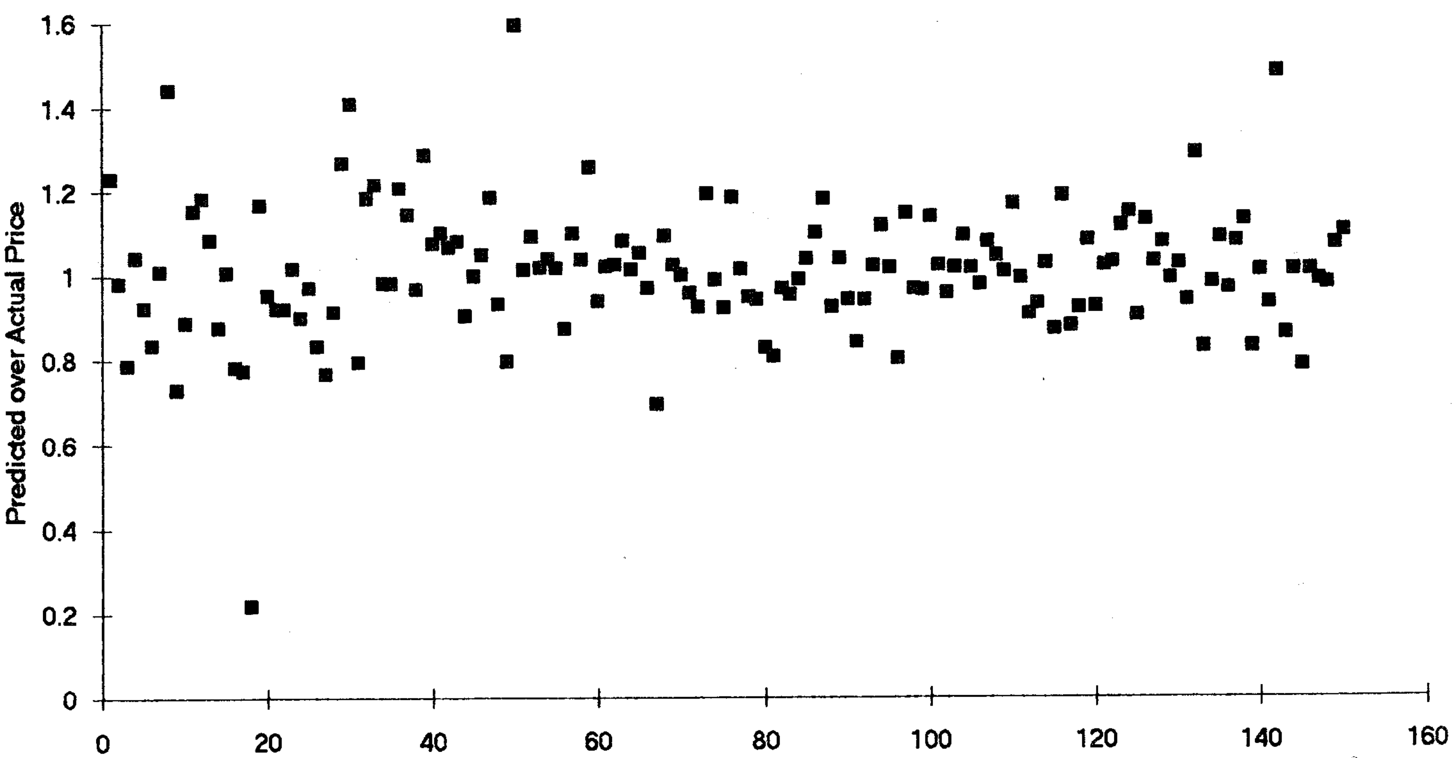

Figure 5.9 Predicted over real price for Thompson-Okanagan Region

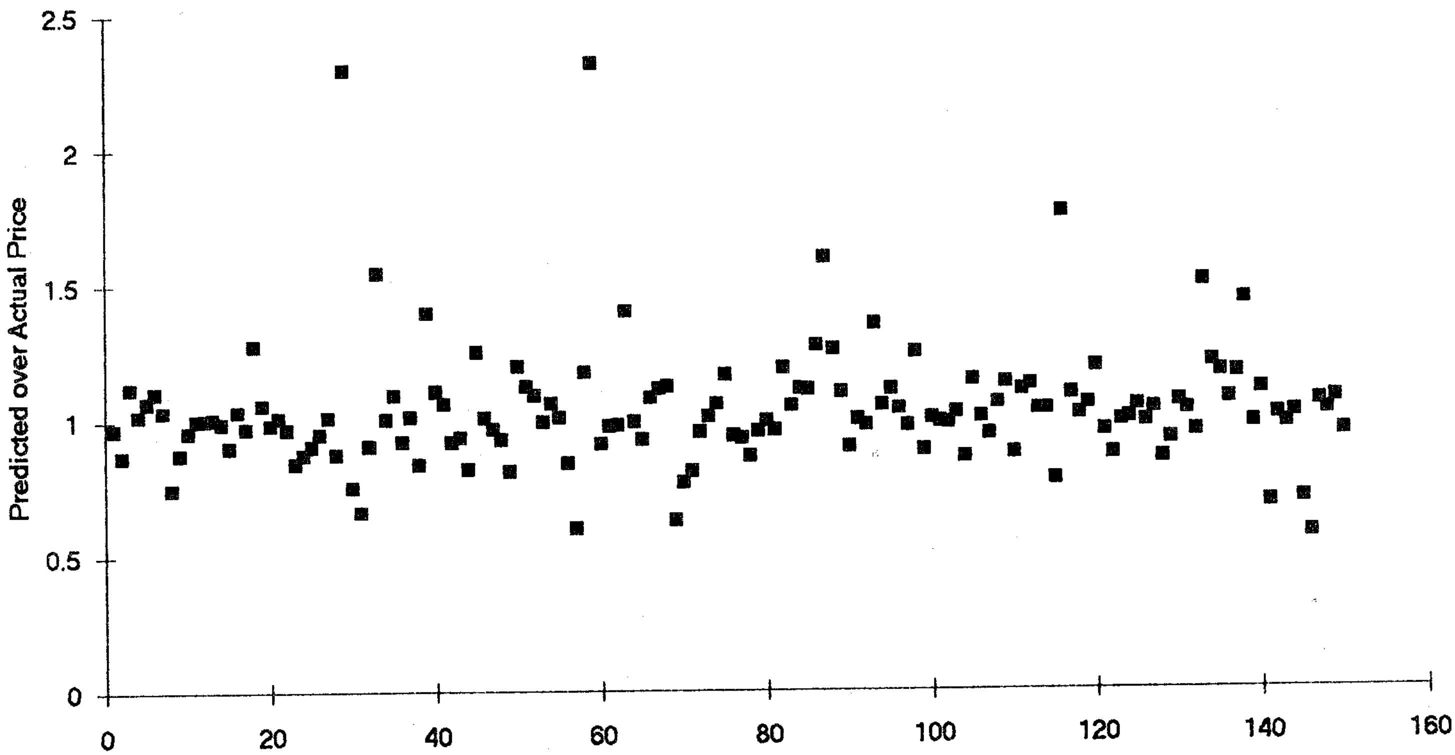

Figure 5.10 Predicted over real price for Kootenay Region

The results appear quite reasonable, with all of the estimated regional prices converging to their actual values (converging around the value one). On average for the regions, the model showed a rough accuracy of about 80 percent (varying between 1.2 and 0.8). There were only a few extreme outliers found in the four of the five regions over the 150 simulations, with Peace River being the excluded region.

The Central Interior Region exhibited two cases in which price was underestimated, with the more significant estimate being approximately 20 percent of actual price. Only two or three overestimates were found, with the largest being greater than two times the actual price. These extreme estimates are most likely a result of prices being generated from the kinked regional demand curves. Since prices along the inelastic portion of the demand curve are based on other trading regions' prices, and there is a significant jump from the upper to lower curves, situations can possibly arise in which the price may appear overly high or low. An example of this would be where a region produces excess production and its surrounding regions have a significant excess demand. As a result the excess supply will flow to the surrounding regions such that the region will consume at its base requirement and have price generated from the other regions prices. A jump in that region's price level would be observed, as compared to if the regions had a strictly linear demand curves (no inelastic portion). Since the regressions used are based on a linear fit, the results may appear to be an extreme over or underestimation in price.[31]

From a policy perspective, the implications of an overestimation in price are more severe than an underestimation. If price is severely overestimated, and the observation falls into the category in which a Wide-Spread Drought payment must be made (i.e., the estimated price exceeding the price trigger level and the actual supply falls below the guarantee level), then it can become quite expensive for the agency supplying the compensation.

The Cariboo-Chilcotin Region only had one notable outlier in its price estimates, where the estimated price exceeded the actual by approximately 60 percent. The Thompson-Okanagan Region had one extreme underestimate, where estimated price was approximately 20 percent of the actual and two overestimates of approximately 50 percent over the actual. The Kootenay Region exhibited the most number of outliers. In over approximately five cases, the estimated price exceeded the actual by more than 50 percent with the largest being more than 100 percent. This can be explained by the fact that the Kootenay Region is significantly isolated from the rest of the province, given the huge transport costs especially beyond the Thompson-Okanagan Region. As a result the price in the Kootenay Region may have relatively larger fluctuations in price than the other regions.[32]

---

[31] An observed overestimation may result if the region has excess supply like in the example above, while underestimation may occur in the reverse if the region is in excess demand.

[32] Due to the large transport costs to the rest of the province the Kootenay Region does almost all of its trade with the Thompson-Okanagan Region; therefore, the Kootenay Region may

As stated previously, the purpose of this thesis was to develop a pricing model for forage. With this model, estimates of forage prices can be made and applied directly to the problem of crop insurance. Forage prices are a necessary component in a WSD insurance scheme since they are used to determine the level of payments, and estimates are needed as prices are non-observable in the real world. The way in which this model would be applied to the WSD insurance scheme will be described.

Given the results from the regressions of prices on quantities (assuming that the 'Base Case Scenario' parameters were used), the regression equations can be used to obtain current price estimates. The Ministry of Agriculture (the insurers) would substitute the current regional production levels for forage back into each of the regression equations to obtain regional price estimates. Given these regional price estimates and their associated quantity levels, the insurers could determine the indemnities (if any) owed to the farmers under the insurance scheme. As described previously, for a region to qualify for a payment under this scheme, actual supply must fall below the guarantee level and price must exceed a price trigger level.

The following graph, Figure 5.11, provides a graphical description of payments under the WSD insurance scheme. The results from the 150 simulations under the 'Base Case Scenario' for the Peace River Region are shown in the plots of predicted price on supply levels. Note that the price trigger[33] and guarantee level[34] are represented. Payment levels are equal to the quantity coverage (amount below the guarantee level) multiplied by the price covered (amount above the price trigger). The points in the top left quadrant represent Wide-Spread Drought payments, where supply falls below the guarantee level and the estimated price exceeds the trigger level.

---

have less price stability in extreme times with only one main trading path.

[33] This value was chosen to be 20% above the average price level.

[34] This value was chosen to be 80% of average production.

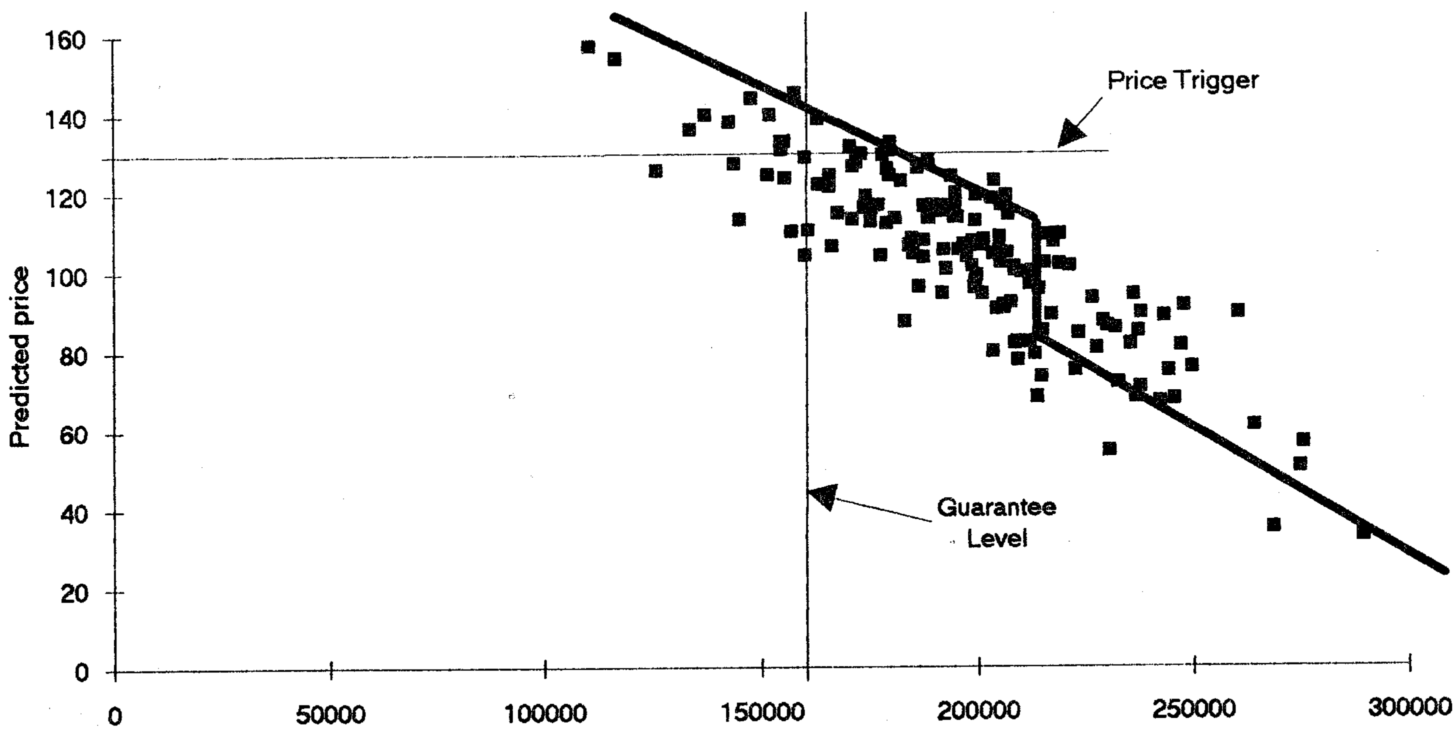

Figure 5.11 Peace River Region (Base Case Scenario)

The results from this graph are useful in that the insurers can determine the frequency and cost of the program. In 150 simulations (years), the number of times in which a payment is made to the Peace River Region can be ascertained by the number of observations in the top left quadrant. The cost of these payments can be determined by the amount by which the observation exceeds the price trigger and falls below the guarantee level. Note that both the frequency and level of payments are of significant importance to the viability of the program.[35]

[35] Both the frequency and levels will change as the price trigger and guarantee levels are changed.

Chapter 6 : Conclusions

During times of wide-spread drought, forage producers and consumers, in particular those in British Columbia, may experience drastic increases in forage prices. It had been proposed that a method be devised in which farmers could insure themselves against this price-risk associated with these types of natural disasters. The Wide-Spread Drought insurance scheme was proposed by the Crop Insurance Branch at British Columbia Ministry of Agriculture, Fisheries, and Foods in order to deal with this issue. This insurance scheme did however, require the use of a model to estimate forage prices based on regional supplies of forage, since prices were non-observable. The first portion of this thesis developed a model to estimate forage prices. The second portion showed an application to British Columbia forage production, with a description of its use in a Wide-Spread Drought insurance scheme.

Aside from the huge limitation on availability of data and vast complexities of the model, some reasonable results were obtained. Under the 'Base Case Scenario' regression analysis was used to form a relationship between changes in regional production levels and changes in regional price levels. From the regressions, the R-squared values and t-statistics all appeared quite reasonable. The accuracy of the model in estimating prices was checked by graphically representing predicted prices over real prices. With the exception of a few extreme values, the model worked quite well in price estimation.

Results from two other sloped scenarios were compared with those of the Base Case in order to determine the models sensitivity to changes in the upper slopes of the regional demand curves. Log-linear regressions were performed so that the changes in quantities and the effects on the price levels could be expressed in percentages. It was assumed that increasing the slopes would make the model more price sensitive and lowering the slopes, less price sensitive. The results validated this assumption as the 'Steep Slope Scenario' exhibited a greater change in regional price estimates above normal levels given an equivalent percentage shortfall in production below normal levels than the 'Base

Case Scenario'. Note that this was true for all but the Cariboo-Chilcotin Region, in which price was only one percent less responsive than in the 'Base Case Scenario'. As well, the 'Flat Slope Scenario' exhibited a smaller change in regional price estimates above normal levels given an equivalent percentage shortfall in production below normal levels than the 'Base Case Scenario'.

The model developed in the thesis shows considerable promise in its applicability to real world problems, in particular to the problem of crop insurance. When applied to British Columbia forage production, it did reasonably well in estimating forage prices and responded quite well to changes in the slopes of the regional demand curves. However, the accuracy of these results is questionable as some of the necessary data was unavailable. In particular, there was no forage consumption or price data available to parameterize the demand curves. As a result, approximations to these curves had to be made. There was no series data available on forage yields to obtain variance/covariance values for the regions, therefore, these values were obtained by drawing relationships between variability in precipitation and variability in yields. There was also limited availability of data on average regional yield levels, amount of seeded acreage used, and feed requirements per type of livestock.

In order for the results to ultimately be useful for price estimation purposes, all of the data requirements must be met. Without this complete information, any conclusions drawn regarding the results may be strictly hypothetical.

Bibliography

Agriculture Canada (1986), <u>Sheep Production and Marketing</u>, Ottawa.

Ardeni, P.G. (1989), "*Does the Law of One Price Really Hold for Commodity Prices?*", American Journal of Agricultural Economics, August, p.661-669.

Aumack, Ken. (Williams Lake), Jim Forbes (Dawson Creek), Denise McLean (Vanderhoof), Mike Momberg (100 Mile House), and Ted Moore (Kamloops). Average regional forage price estimates were obtained from the aforementioned forage specialists, 1994.

Baffes, J. (1991), "*Some Further Evidence on the Law of One Price : The Law of One Price Still Holds*", American Journal of Agricultural Economics, November, P.1264-73.

Beames, R.M. (Dept. of Animal Science, UBC), Ron Charles, Bob France, Wayne Hadow, and Marc Robbins (British Columbia Ministry of Agriculture, Fisheries, and Foods). Personal consultation was made with the aforementioned agriculture specialists, 1994.

Beckmann, M.J. (1985), "*Competitive Mill and Uniform Pricing*", in Norman, G., <u>Spatial Pricing and Differentiated Markets</u>, p.52-64.

Blake, Martin J., and Tom Clevenger (1984), "*A Linked Annual and Monthly Model for Forecasting Alfalfa Hay Prices*", Western Journal of Agricultural Economics, 9(1), p.195-199.

Blank, S.C., and H.W. Ayer (1987), "*Government Policy Cross Effects : The Cotton and Dairy Programs' Influence of Alfalfa Hay Markets*", Agribusiness (3), p.385-392.

British Columbia Ministry of Agriculture, Fisheries, and Foods, Crop Insurance Branch. Personal consultation and information on yields.

British Columbia Ministry of Agriculture, Fisheries, and Foods (1994), "*Hay Storage Options*", Victoria.

Brook, Anthony., David Kendrick, and Alexander Meeraus (1988), <u>Gams : A User's Guide</u>. Redwood City, California : The Scientific Press.

<u>California Alfalfa Hay : Processing, Transporting and Pricing</u>. (1976), Division of Agricultural Sciences, University of California, Davis.

Harris, Curtis C., and Mehrzad Nadji (1987), <u>Regional Economic Modelling in Theory and in Practice</u>. Lanham, MD : University Press of America.

Judge, G. George, et al. (1988), <u>Introduction to the Theory and Practice of Econometrics</u>. Toronto : John Wiley and Sons, p.494-496.

Keay, Roger G. (1991), <u>Cow-Calf Farm Business Analysis</u>, Summary Report, British Columbia Ministry of Agriculture, Fisheries, and Food.

Konyar, K., and K. Knapp (1988), "*Market Analysis of Alfalfa Hay : California Case*", Agribusiness (4), p.271-284.

Konyar, K., and K. Knapp (1990), "*Dynamic Regional Analysis of the California Alfalfa Market with Government Policy Impacts*", Western Journal of Agricultural Economics, 15(1), p. 22-32.

Liew, Chong K., and Jae K. Shim (1978), "*A Spatial Equilibrium Model : Another View*", Journal of Urban Economics (5), p.526-534.

McConughy, J. and L. (Prince George), Pickseed Canada Inc. (Dawson Creek), Risky Creek (Williams Lake), and Ray Schmere (Peace River, Alta). Transportation cost quotes were obtained from the aforementioned trucking companies, 1994.

Meyer, G.L., and J.F. Yanagida (1981), "*A Methodology of Combining Annual Econometric Forecasts with Quarterly ARIMA Forecasts*", Contributed Paper, Western Agricultural Economics Association Meeting, Lincoln, Nebraska, July 19-21.

Meyer, G.L., and J.F. Yanagida (1984), "*Combining Annual Econometric Forecasts with Quarterly ARIMA Forecasts : A Heuristic Approach*", Western Journal of Agricultural Economics, 9(1), p.200-206.

Ministry of Tourism (1987), British Columbia Road Map and Parks Guide, Victoria.

National Research Council (1989), "*Nutrient Requirements of Domestic Animals*", United States.

Ross, C.V. (1989), Sheep-Production and Management. Englewood Cliffs, New Jersey : Prentice Hall.

Soder, E.M. (1976), "*Field Loss Aspects*", Alfalfa Seminar, Kamloops, B.C.

Statistics Canada (1987-1991), #22-002 Seasonal, November Estimates of Principal Field Crops, Canada, Vol. 66-71, No. 8, Ottawa.

Statistics Canada (1990), #22-201, Annual Statistics, Ottawa.

Statistics Canada (1990), #22-201, Grain Trade of Canada, Ottawa.

Statistics Canada (1991), #95-393D, Agricultural Profile of British Columbia : 1991 Census of Agriculture, Ottawa.

Takayama, T., and W.C. Labys (1986), "*Spatial Equilibrium Analysis*", Handbook of Regional and Urban Economics, Vol. 1, p. 171-199.

White, Kenneth J. (1993), Shazam : Econometrics Computer Program 7.0, User's Reference Manual. Montreal : McGraw-Hill, p.392.

Willet, K. (1983), "*Single- and Multi-Commodity Models of Spatial Equilibrium in a Linear Programming Framework*", Canadian Journal of Agricultural Economics, July, p.205-222.

Appendix 1

Data

Table 1

Regional Livestock Numbers			
Region :	Cows (Beef/Dairy)	Horses (Ponies)	Sheep (Lambs)
Peace River	46,873	6,163	10,898
Central Interior	36,552	3,634	8,219
Cariboo-Chilcotin	51,829	4,256	6,486
Thompson-Okanagan	87,927	11,265	15,068
Kootenay	17,344	2,207	1,450

Table 2

Regional Base Herd Feed Requirement					
Region :	Feeding Period (days)	Regional Cow Feed Required in tons (using 0.02 tons/day per cow)	Reg.Horse Feed Required in tons (0.01 tons/day per horse)	Reg.Sheep Feed Required in tons (0.003 tons/day per sheep)	Total Regional Base Herd Feed Require-ment
Peace River	210	196,867	12,942	6,866	216,675
Central Interior	205	149,863	7,450	5,055	162,368
Cariboo-Chilcotin	190	196,950	8,086	3,697	208,733
Thompson-Okanagan	130	228,610	14,645	5,877	249,132
Kootenay	170	58,970	3,752	740	63,462

Table 3

Yield Per Acre for Forage Crops			
Region :	Yield / Acre (from Keay, 1991)	Yield / Acre (from B.C. Ministry of Agr. Crop Ins.)	Actual Yield per Acre values used in the study
Peace River	1.4	1.092 (N.Peace) 1.477 (S.Peace)	1.2
Central Interior	1.7	1.656	1.7
Cariboo-Chilcotin	2.1	1.578	1.58
Thompson-Okanagan	2.8	2.885	2.8
Kootenay	2.8	2.450	2.8

Table 4

Seeded Acreage and Average Regional Production				
Region :	Alfalfa & Alfalfa Mixtures	All other Tame Hay	Total Seeded Acreage (less 10-20% for double-counting)	Average Regional Forage Production (less 20% for model calibration)
Peace River	101,529	152,811	203,472 (less 20%)	195,333
Central Interior	49,916	108,449	142,529 (less 10%)	193,839
Cariboo-Chilcotin	39,935	58,424	88,523 (less 10%)	111,893
Thompson-Okanagan	111,122	46,751	142,086 (less 10%)	318,272
Kootenay	28,196	10,457	34,788 (less 10%)	77,925

Table 5

Coefficient of Variation/Correlation Coefficient Matrix for Regional Precipitation					
Region :	Peace River	Central Interior	Cariboo-Chilcotin	Thompson-Okanagan	Kootenay
Peace River	0.5894	0.5450	0.4581	0.3634	0.2957
Central Interior	0.5450	0.4350	0.8297	0.5259	0.3698
Cariboo-Chilcotin	0.4581	0.8297	0.5165	0.6207	0.4373
Thompson-Okanagan	0.3634	0.5259	0.6207	0.4878	0.6528
Kootenay	0.2957	0.3698	0.4373	0.6528	0.4522

Table 6

Cholesky's Decomposed Matrix (for production)					
Region :	Peace River	Central Interior	Cariboo-Chilcotin	Thompson-Okanagan	Kootenay
Peace River	34,539	0	0	0	0
Central Interior	4,136	24,956	0	0	0
Cariboo-Chilcotin	2,383	3,980	16,706	0	0
Thompson-Okanagan	5,078	6,607	6,703	45,332	0
Kootenay	938	1,033	1,059	1,715	10,283

Table 7

Regional Transportation Costs Matrix (dollars per ton)					
Region :	Peace River	Central Interior	Cariboo-Chilcotin	Thompson-Okanagan	Kootenay
Peace River	7	42	48	61	78
Central Interior	42	13	25	43	61
Cariboo-Chilcotin	48	25	13	22	56
Thompson-Okanagan	61	43	22	13	42
Kootenay	78	61	56	42	13
Rates : < 50 miles ($0.135 ton/loaded mile); 50 - 101 miles ($0.13); 101 - 350 miles ($0.12); 350 - 450 miles ($0.11); > 450 miles ($0.10)					

Table 8

Regional Storage Costs (dollars per ton)			
Region :	Value Lost Due To Spoilage	Average Regional Forage Price*	Regional Storage Cost
Peace River	0.25 %	72	18
Central Interior	0.25 %	83	21
Cariboo-Chilcotin	0.25 %	65	16
Thompson-Okanagan	0.25 %	89	22
Kootenay	0.25 %	82	21
* Average price obtained from an average of the regional export and import prices			

Appendix 2

Sensitivity Analysis

Log-Linear regressions for 'Steep Slope Scenario' :

%dPPE=-1.22%dQPE-0.44%dQCE-0.33%dQCA-0.44%dQTH-0.08%dQKO

%dPCE=-0.77%dQPE-1.17%dQCE-0.58%dQCA-1.18%dQTH-0.30%dQKO

%dPCA=-0.39%dQPE-0.58%dQCE-0.34%dQCA-0.96%dQTH-0.19%dQKO

%dPTH=-0.55%dQPE-0.74%dQCE-0.52%dQCA-1.34%dQTH-0.23%dQKO

%dPKO=-0.45%dQPE-0.35%dQCE-0.47%dQCA-1.06%dQTH-0.62%dQKO

Log-Linear regressions for 'Flat Slope Scenario' :

%dPPE=-1.05%dQPE-0.41%dQCE-0.24%dQCA-0.30%dQTH-0.03%dQKO

%dPCE=-0.61%dQPE-0.99%dQCE-0.39%dQCA-0.76%dQTH-0.15%dQKO

%dPCA=-0.26%dQPE-0.50%dQCE-0.27%dQCA-0.68%dQTH-0.12%dQKO

%dPTH=-0.31%dQPE-0.63%dQCE-0.34%dQCA-1.14%dQTH-0.11%dQKO

%dPKO=-0.10%dQPE-0.32%dQCE-0.12%dQCA-0.53%dQTH-0.35%dQKO

Appendix 3

```
* GAMS file to run the spatial equilibrium model

* DEFINING THE FILE TO WRITE TO

 FILE outt /outt.dat/;

 FILE con / con.dat/;

 FILE tostor /tostor.dat/;

 FILE storin /storin.dat/;

* "SET(S)" DECLARES THE SETS (DOMAIN) OVER WHICH THE PARAMETERS
* AND VARIABLES ARE DEFINED (I.E. VALUES FOR EACH REGION).  PAGES
* 5 TO 7 OF A GUIDE TO USING GAMS.

 SET
  R REGIONS
   /PEACE
    CENTRAL
    CARIBOO
    THOMPS
    KOOTEN
    STORAGE/;

 SET
  U ITERATIONS
   /U1*U150/;

 SET
  Y REPETITIONS FOR SOLVING PRICE
   /Y1*Y7/;

* PARAMETER (FOR ENTERING PARAMETER LISTS, I.E. VECTORS)
* TABLE (FOR TWODIMENSIONAL TABLES, I.E. MATRICES)
* SCALAR (FOR SCALARS, I.E. SINGLE ELEMENTS, CONSTANTS)

 PARAMETER ALU(R) UPPER INTERCEPT OF DEMAND CURVES
  /PEACE 225
   CENTRAL 154
   CARIBOO 194
   THOMPS 239
   KOOTEN 239
   STORAGE 78/;

 PARAMETER ALD(R) LOWER INTERCEPT OF DEMAND CURVES
  /PEACE 251
   CENTRAL 180
   CARIBOO 220
   THOMPS 265
   KOOTEN 91
   STORAGE 0/;

 PARAMETER BU(R) UPPER SLOPE OF DEMAND CURVES
  /PEACE 0.00052
   CENTRAL 0.00044
   CARIBOO 0.00051
   THOMPS 0.00055
   KOOTEN 0.0027
   STORAGE 0.000189/;

 PARAMETER BL(R) LOWER SLOPE OF DEMAND CURVES
  /PEACE 0.00075
   CENTRAL 0.00075
   CARIBOO 0.00075
   THOMPS 0.00075
```

```
   KOOTEN 0.00075
   STORAGE 0/;

* THE PARAMETER CBAR REPRESENTS THE CONSUMPTION REQUIREMENT, ie THE
* VERTICAL PORTION OF THE DEMAND CURVE.  IT IS REPRESENTED BY NUMBER OF
* COWS MULTIPLIED BY THE FEED REQUIREMENT PER COW.

 PARAMETER CB(R) CONSUMPTION REQUIREMENT
  /PEACE 216675
   CENTRAL 162368
   CARIBOO 208734
   THOMPS 249131
   KOOTEN 63461
   STORAGE 413136/;

*NOTE BELOW THAT THE MEAN VALUES FOR THESE INDEPENDENT RANDOM QUANTITIES IS
*ACCOUNTED FOR BY ADDING A VECTOR OF MEAN VALUES M TO THE VECTOR OR
*CORRELATED QUANTITIES, OBTAINED BY MULTIPLYING BY THE CHOLESKY MATRIX.
*THE VARIANCES FOR THE INDEP's IS ACCOUNTED FOR IN THE CHOLESKY
*DECOMPOSITION MATRIX (i.e. *FROM THE COVARIANCE MATRIX)

 PARAMETER IND(R) INDEPENDENT RANDOM QUANTITY DRAWS;
  IND('PEACE')=NORMAL(0,1);
  IND('CENTRAL')=NORMAL(0,1);
  IND('CARIBOO')=NORMAL(0,1);
  IND('THOMPS')=NORMAL(0,1);
  IND('KOOTEN')=NORMAL(0,1);

 PARAMETER M(R) MEAN QUANTITIES PRODUCED
   /PEACE 195333
    CENTRAL 193839
    CARIBOO 111893
    THOMPS 318272
    KOOTEN 77925/;

 ALIAS (R,RP);

*R IS ROWS AND RP IS COLUMNS

 TABLE CHOL(R,RP) CHOLESKY DECOMPOSITION MATRIX FOR THE RANDOM QUANTITIES

            PEACE     CENTRAL   CARIBOO     THOMPS    KOOTEN
PEACE       34539       0         0           0         0
CENTRAL     4136       24956      0           0         0
CARIBOO     2383       3980      16706        0         0
THOMPS      5078       6607      6703       45332       0
KOOTEN      938        1033      1059        1715      10283;

 PARAMETER QP(R) CORRELATED RANDOM QUANTITIES EXCLUDING CURRENT STORAGE;
  QP('PEACE')=SUM(RP,CHOL('PEACE',RP)*IND(RP))+M('PEACE');
  QP('CENTRAL')=SUM(RP,CHOL('CENTRAL',RP)*IND(RP))+M('CENTRAL');
  QP('CARIBOO')=SUM(RP,CHOL('CARIBOO',RP)*IND(RP))+M('CARIBOO');
  QP('THOMPS')=SUM(RP,CHOL('THOMPS',RP)*IND(RP))+M('THOMPS');
  QP('KOOTEN')=SUM(RP,CHOL('KOOTEN',RP)*IND(RP))+M('KOOTEN');

 QP('PEACE')=MAX(QP('PEACE'),0);
 QP('CENTRAL')=MAX(QP('CENTRAL'),0);
 QP('CARIBOO')=MAX(QP('CARIBOO'),0);
 QP('THOMPS')=MAX(QP('THOMPS'),0);
 QP('KOOTEN')=MAX(QP('KOOTEN'),0);

 PARAMETER PRICE(R) PRICE VALUES CALCULATED INSIDE THE LOOP;

 PARAMETER S1(R,U) CURRENT STOR ADDED TO CURRENT PROD IN RUN2 FROM SERIES 1;

 PARAMETER S2(R,U) CURRENT STOR ADDED TO CURRENT PROD IN RUN3 FROM SERIES 2;
```

```
PARAMETER S3(R,U) CURRENT STOR ADDED TO CURRENT PROD IN RUN4 FROM SERIES 3;

PARAMETER S4(R,U) CURRENT STOR ADDED TO CURRENT PROD IN RUN5 FROM SERIES 4;

PARAMETER S5(R,U) CURRENT STOR ADDED TO CURRENT PROD IN RUN6 FROM SERIES 5;


* NORMALLY TABLES ARE DEFINED OVER TWO DIFFERENT SETS.  EXAMPLE,
* IN THE STANDARD LP PROBLEM, THE TWO SETS ARE ACTIVITIES
* (COLUMNS) AND INPUTS (ROWS) OF THE COEFFICIENT MATRIX.  IN THE
* PRESENT 3 AGENT PROBLEM, THE R SET IS USED TO DEFINE BOTH THE
* COLUMNS AND THE ROWS OF THE TRANSPORTATION COST MATRIX.  NOTE,
* WE HAVE TO USE THE "ALIAS" COMMAND (P.35) TO GIVE THE R SET
* ANOTHER NAME.  WE WILL CALL THE NEW NAME RP (I.E. R 'PRIME').
* THE FIRST SCRIPT IN THE TABLE REFERS TO THE ROWS AND THE
* SECOND REFERS TO THE COLUMNS (P.26).

 TABLE A(R,RP) TRANSPORTATION COSTS BETWEEN REGIONS

           PEACE     CENTRAL    CARIBOO     THOMPS    KOOTEN    STORAGE
PEACE        7         42         48          61        78         18
CENTRAL     42         13         25          43        61         21
CARIBOO     48         25         13          22        56         16
THOMPS      61         43         22          13        42         22
KOOTEN      78         61         56          42        13         21
STORAGE     18         21         16          22        21          0;

* VARIABLES CAN BE EITHER POSITIVE, NEGATIVE, INTEGER, BINARY, OR
* FREE (CAN TAKE ON ANY VALUE).  GAMS DEFAULTS TO FREE VARIABLES.

 VARIABLES

  C(R) CONSUMPTION IN EACH REGION (AGENT)
  TR(R,RP) AMOUNT TRANSPORTED BETWEEN REGIONS
  W TOTAL WELFARE
  IA(R) INDICATOR FOR C LESS THAN CBAR
  IB(R) INDICATOR FOR C GREATER THAN CBAR
  POSITIVE VARIABLES C,TR,IA,IB;

* EQUATIONS NEED TO FIRST BE DESCRIBED (DECLARED) AND THEN
* DEFINED.  WHEN DEFINING EQUATIONS, THEIR NAMES ARE FOLLOWED BY
* TWO DOTS.  =E= DENOTED EQUAL TO, =G= GREATER THAN EQUAL TO, AND
* =L= LESS THAN EQUAL TO.

 EQUATIONS

  WELFARE OBJECTIVE FUNCTION
  CONSUM(R) CONSUMPTION CONSTRAINTS
  QUANT(R) QUANTITY CONSTRAINTS
  STOREA(R) STORAGEA CONSTRAINT
  STOREB(R) STORAGEB CONSTRAINT
  ICONA(R) ICONSTRA CONSTRAINT
  ICONB(R) ICONSTRB CONSTRAINT
  ICONC(R) ICONSTRC CONSTRAINT
  ICOND(R) ICONSTRD CONSTRAINT
  ICONE(R) ICONSTRE CONSTRAINT;

  WELFARE.. W=E=SUM(R,IA(R)*(ALU(R)*C(R)-0.5*BU(R)*C(R)*C(R))
                +IB(R)*(ALU(R)*CB(R)-0.5*BU(R)*CB(R)*CB(R)
                +(C(R)-CB(R))*(ALD(R)-BL(R)*(C(R)+CB(R))/2)))
                 -SUM((R,RP),A(R,RP)*TR(R,RP));
  CONSUM(R).. C(R)=L=SUM(RP,TR(R,RP));
  QUANT(R).. QP(R)=G=SUM(RP,TR(RP,R));
  STOREA(R).. TR('STORAGE',R)=G=0;
  STOREB(R).. TR(R,'STORAGE')=E=0;
```

```
  ICONA(R).. IA(R)*(C(R)-CB(R))=L=0;
  ICONB(R).. IB(R)*(C(R)-CB(R))=G=0;
  ICONC(R).. IA(R)+IB(R)=E=1;
  ICOND(R).. IA(R)=L=1;
  ICONE(R).. IB(R)=L=1;

* .L DESIGNATES A STARTING VALUE AND .FX DENOTES A FIXED
* VARIABLE, (P. 47 TO 48).  WE WILL USE THE AUTARKY SOLUTION AS OUR
* STARTING VALUES (NO TRADE).  THE DEFAULT STARTING VALUE IS 0.

  C.L(R) = QP(R);

* "MODEL' IS USED TO NAME THE MODEL AND TO IDENTIFY THE EQUATIONS
* WHICH IT INCUDES, (P.11 TO 12).  WE CALL OUR MODEL 'VERSION1' AND
* TELL GAMS TO INCLUDE ALL OF THE EQUATIONS, I.E. WELFARE,
* CONSUMPTION AND QUANTITY PRODUCED.

 MODEL VERSION1 /ALL/;

* HERE IS A TITLE FOR THE PRICES AND QUANTITES SENT TO outt.dat

 PUT outt
 PUT "      RESULTS FROM GAMS OUTPUT"/
 PUT "-----------------------------------"//
 PUT "    QP1     P1     QP2     P2     QP3     P3     QP4     P4     QP5     P5"/;

 PUT con
 PUT " CONSUMPTION"/
 PUT "-----------------------------------"//
 PUT " CONPEA   CONCEN    CONCAR    CONTHO   CONKOO  "/;

 PUT tostor
 PUT " ALLOCATION TO STORAGE"/
 PUT "-----------------------------------"//
 PUT "  PRICESTO  STOPEA    STOCEN    STOCAR   STOTHO     STOKOO   "/;

 PUT storin
 PUT "STORAGE ADDED TO CURRENT PRODUCTION"/
 PUT "-----------------------------------"//
 PUT " STOPEA    STOCEN    STOCAR   STOTHO     STOKOO   "/;

 OPTION SEED = 2576;

*CREATE THE LOOPS TO GENERATE THE PRICE DISTRIBUTION OVER SET U
*********START OF FIRST LOOP

 LOOP(U,

 IND('PEACE')=NORMAL(0,1);
 IND('CENTRAL')=NORMAL(0,1);
 IND('CARIBOO')=NORMAL(0,1);
 IND('THOMPS')=NORMAL(0,1);
 IND('KOOTEN')=NORMAL(0,1);

 QP('PEACE')=SUM(RP,CHOL('PEACE',RP)*IND(RP))+M('PEACE');
 QP('CENTRAL')=SUM(RP,CHOL('CENTRAL',RP)*IND(RP))+M('CENTRAL');
 QP('CARIBOO')=SUM(RP,CHOL('CARIBOO',RP)*IND(RP))+M('CARIBOO');
 QP('THOMPS')=SUM(RP,CHOL('THOMPS',RP)*IND(RP))+M('THOMPS');
 QP('KOOTEN')=SUM(RP,CHOL('KOOTEN',RP)*IND(RP))+M('KOOTEN');

 QP('PEACE')=MAX(QP('PEACE'),0);
 QP('CENTRAL')=MAX(QP('CENTRAL'),0);
 QP('CARIBOO')=MAX(QP('CARIBOO'),0);
 QP('THOMPS')=MAX(QP('THOMPS'),0);
 QP('KOOTEN')=MAX(QP('KOOTEN'),0);
```

```
 C.L(R)=QP(R);

* "SOLVE" INDICATES : (A) THE MODEL TO BE SOLVED; (B) THE
* DIRECTION OF SOLUTION (MAX/MIN); (C) THE NAME OF THE OBJECTIVE
* VARIABLE; (D) THE SOLUTION PROCEDURE TO BE USED.

 SOLVE VERSION1 MAXIMIZING W USING NLP;

 S1('PEACE',U) = TR.L('STORAGE','PEACE');
 S1('CENTRAL',U) = TR.L('STORAGE','CENTRAL');
 S1('CARIBOO',U) = TR.L('STORAGE','CARIBOO');
 S1('THOMPS',U) = TR.L('STORAGE','THOMPS');
 S1('KOOTEN',U) = TR.L('STORAGE','KOOTEN');

********END OF FIRST LOOP
);

 OPTION SEED = 3367;

*********START OF LOOP 2

 LOOP(U,

 IND('PEACE')=NORMAL(0,1);
 IND('CENTRAL')=NORMAL(0,1);
 IND('CARIBOO')=NORMAL(0,1);
 IND('THOMPS')=NORMAL(0,1);
 IND('KOOTEN')=NORMAL(0,1);

 QP('PEACE')=SUM(RP,CHOL('PEACE',RP)*IND(RP))+M('PEACE')+S1('PEACE',U);
 QP('CENTRAL')=SUM(RP,CHOL('CENTRAL',RP)*IND(RP))+M('CENTRAL')+S1('CENTRAL',U);
 QP('CARIBOO')=SUM(RP,CHOL('CARIBOO',RP)*IND(RP))+M('CARIBOO')+S1('CARIBOO',U);
 QP('THOMPS')=SUM(RP,CHOL('THOMPS',RP)*IND(RP))+M('THOMPS')+S1('THOMPS',U);
 QP('KOOTEN')=SUM(RP,CHOL('KOOTEN',RP)*IND(RP))+M('KOOTEN')+S1('KOOTEN',U);

 QP('PEACE')=MAX(QP('PEACE'),0);
 QP('CENTRAL')=MAX(QP('CENTRAL'),0);
 QP('CARIBOO')=MAX(QP('CARIBOO'),0);
 QP('THOMPS')=MAX(QP('THOMPS'),0);
 QP('KOOTEN')=MAX(QP('KOOTEN'),0);

 C.L(R)=QP(R);

 SOLVE VERSION1 MAXIMIZING W USING NLP;

 S2('PEACE',U)=TR.L('STORAGE','PEACE');
 S2('CENTRAL',U)=TR.L('STORAGE','CENTRAL');
 S2('CARIBOO',U)=TR.L('STORAGE','CARIBOO');
 S2('THOMPS',U)=TR.L('STORAGE','THOMPS');
 S2('KOOTEN',U)=TR.L('STORAGE','KOOTEN');

*********END LOOP 2
 );

 OPTION SEED =2107

******START LOOP 3
 LOOP(U,

 IND('PEACE')=NORMAL(0,1);
 IND('CENTRAL')=NORMAL(0,1);
 IND('CARIBOO')=NORMAL(0,1);
 IND('THOMPS')=NORMAL(0,1);
 IND('KOOTEN')=NORMAL(0,1);

 QP('PEACE')=SUM(RP,CHOL('PEACE',RP)*IND(RP))+M('PEACE')+S2('PEACE',U);
```

```
 QP('CENTRAL')=SUM(RP,CHOL('CENTRAL',RP)*IND(RP))+M('CENTRAL')+S2('CENTRAL',U);
 QP('CARIBOO')=SUM(RP,CHOL('CARIBOO',RP)*IND(RP))+M('CARIBOO')+S2('CARIBOO',U);
 QP('THOMPS')=SUM(RP,CHOL('THOMPS',RP)*IND(RP))+M('THOMPS')+S2('THOMPS',U);
 QP('KOOTEN')=SUM(RP,CHOL('KOOTEN',RP)*IND(RP))+M('KOOTEN')+S2('KOOTEN',U);

 QP('PEACE')=MAX(QP('PEACE'),0);
 QP('CENTRAL')=MAX(QP('CENTRAL'),0);
 QP('CARIBOO')=MAX(QP('CARIBOO'),0);
 QP('THOMPS')=MAX(QP('THOMPS'),0);
 QP('KOOTEN')=MAX(QP('KOOTEN'),0);

 C.L(R)=QP(R);

 SOLVE VERSION1 MAXIMIZING W USING NLP;

 S3('PEACE',U)=TR.L('STORAGE','PEACE');
 S3('CENTRAL',U)=TR.L('STORAGE','CENTRAL');
 S3('CARIBOO',U)=TR.L('STORAGE','CARIBOO');
 S3('THOMPS',U)=TR.L('STORAGE','THOMPS');
 S3('KOOTEN',U)=TR.L('STORAGE','KOOTEN');

******END LOOP 3
 );

 OPTION SEED = 2688;

*********START LOOP 4

 LOOP(U,

 IND('PEACE')=NORMAL(0,1);
 IND('CENTRAL')=NORMAL(0,1);
 IND('CARIBOO')=NORMAL(0,1);
 IND('THOMPS')=NORMAL(0,1);
 IND('KOOTEN')=NORMAL(0,1);

 QP('PEACE')=SUM(RP,CHOL('PEACE',RP)*IND(RP))+M('PEACE')+S3('PEACE',U);
 QP('CENTRAL')=SUM(RP,CHOL('CENTRAL',RP)*IND(RP))+M('CENTRAL')+S3('CENTRAL',U);
 QP('CARIBOO')=SUM(RP,CHOL('CARIBOO',RP)*IND(RP))+M('CARIBOO')+S3('CARIBOO',U);
 QP('THOMPS')=SUM(RP,CHOL('THOMPS',RP)*IND(RP))+M('THOMPS')+S3('THOMPS',U);
 QP('KOOTEN')=SUM(RP,CHOL('KOOTEN',RP)*IND(RP))+M('KOOTEN')+S3('KOOTEN',U);

 QP('PEACE')=MAX(QP('PEACE'),0);
 QP('CENTRAL')=MAX(QP('CENTRAL'),0);
 QP('CARIBOO')=MAX(QP('CARIBOO'),0);
 QP('THOMPS')=MAX(QP('THOMPS'),0);
 QP('KOOTEN')=MAX(QP('KOOTEN'),0);

 C.L(R)=QP(R);

 SOLVE VERSION1 MAXIMIZING W USING NLP;

 S4('PEACE',U)=TR.L('STORAGE','PEACE');
 S4('CENTRAL',U)=TR.L('STORAGE','CENTRAL');
 S4('CARIBOO',U)=TR.L('STORAGE','CARIBOO');
 S4('THOMPS',U)=TR.L('STORAGE','THOMPS');
 S4('KOOTEN',U)=TR.L('STORAGE','KOOTEN');

********END LOOP 4
 );

 OPTION SEED = 4444;

*******START LOOP 5

 LOOP(U,
```

```
 IND('PEACE')=NORMAL(0,1);
 IND('CENTRAL')=NORMAL(0,1);
 IND('CARIBOO')=NORMAL(0,1);
 IND('THOMPS')=NORMAL(0,1);
 IND('KOOTEN')=NORMAL(0,1);

 QP('PEACE')=SUM(RP,CHOL('PEACE',RP)*IND(RP))+M('PEACE')+S4('PEACE',U);
 QP('CENTRAL')=SUM(RP,CHOL('CENTRAL',RP)*IND(RP))+M('CENTRAL')+S4('CENTRAL',U);
 QP('CARIBOO')=SUM(RP,CHOL('CARIBOO',RP)*IND(RP))+M('CARIBOO')+S4('CARIBOO',U);
 QP('THOMPS')=SUM(RP,CHOL('THOMPS',RP)*IND(RP))+M('THOMPS')+S4('THOMPS',U);
 QP('KOOTEN')=SUM(RP,CHOL('KOOTEN',RP)*IND(RP))+M('KOOTEN')+S4('KOOTEN',U);

 QP('PEACE')=MAX(QP('PEACE'),0);
 QP('CENTRAL')=MAX(QP('CENTRAL'),0);
 QP('CARIBOO')=MAX(QP('CARIBOO'),0);
 QP('THOMPS')=MAX(QP('THOMPS'),0);
 QP('KOOTEN')=MAX(QP('KOOTEN'),0);

 C.L(R)=QP(R);

 SOLVE VERSION1 MAXIMIZING W USING NLP;

 S5('PEACE',U)=TR.L('STORAGE','PEACE');
 S5('CENTRAL',U)=TR.L('STORAGE','CENTRAL');
 S5('CARIBOO',U)=TR.L('STORAGE','CARIBOO');
 S5('THOMPS',U)=TR.L('STORAGE','THOMPS');
 S5('KOOTEN',U)=TR.L('STORAGE','KOOTEN');

********END LOOP 5
 );

 OPTION SEED = 4111;

********START LOOP 6

 LOOP(U,

 IND('PEACE')=NORMAL(0,1);
 IND('CENTRAL')=NORMAL(0,1);
 IND('CARIBOO')=NORMAL(0,1);
 IND('THOMPS')=NORMAL(0,1);
 IND('KOOTEN')=NORMAL(0,1);

 QP('PEACE')=SUM(RP,CHOL('PEACE',RP)*IND(RP))+M('PEACE')+S5('PEACE',U);
 QP('CENTRAL')=SUM(RP,CHOL('CENTRAL',RP)*IND(RP))+M('CENTRAL')+S5('CENTRAL',U);
 QP('CARIBOO')=SUM(RP,CHOL('CARIBOO',RP)*IND(RP))+M('CARIBOO')+S5('CARIBOO',U);
 QP('THOMPS')=SUM(RP,CHOL('THOMPS',RP)*IND(RP))+M('THOMPS')+S5('THOMPS',U);
 QP('KOOTEN')=SUM(RP,CHOL('KOOTEN',RP)*IND(RP))+M('KOOTEN')+S5('KOOTEN',U);

 QP('PEACE')=MAX(QP('PEACE'),0);
 QP('CENTRAL')=MAX(QP('CENTRAL'),0);
 QP('CARIBOO')=MAX(QP('CARIBOO'),0);
 QP('THOMPS')=MAX(QP('THOMPS'),0);
 QP('KOOTEN')=MAX(QP('KOOTEN'),0);

 C.L(R)=QP(R);

SOLVE VERSION1 MAXIMIZING W USING NLP;

* SOLUTION REPORT

* GENERATING THE PRICES

 C.L(R)=ROUND(C.L(R),4);
```

```
 LOOP(R,
      PRICE(R)$(C.L(R) GT CB(R)) = ALD(R)-BL(R)*C.L(R);
      PRICE(R)$(C.L(R) LT CB(R)) = ALU(R)-BU(R)*C.L(R);
      PRICE(R)$(C.L(R) EQ CB(R)) = 0;
       );

 LOOP(Y,
 LOOP((R,RP),
   PRICE(R)$(C.L(R) EQ CB(R) AND TR.L(RP,R) GT 0 AND PRICE(RP)
    NE 0) = PRICE(RP)-A(RP,R);
   PRICE(R)$(C.L(R) EQ CB(R) AND TR.L(R,RP) GT 0 AND PRICE(RP)
    NE 0) = PRICE(RP)+A(R,RP);
    );
    );

* SENDING THE QUANTITIES AND PRICES TO THE FILE outt.dat

  PUT outt;
  PUT QP('PEACE'):12:3 PRICE('PEACE'):9:3 QP('CENTRAL'):12:3
      PRICE('CENTRAL'):9:3 QP('CARIBOO'):12:3 PRICE('CARIBOO'):9:3
      QP('THOMPS'):12:3 PRICE('THOMPS'):9:3 QP('KOOTEN'):12:3
      PRICE('KOOTEN'):9:3/;

  PUT con;
  PUT C.L('PEACE'):12:3 C.L('CENTRAL'):12:3 C.L('CARIBOO'):12:3
      C.L('THOMPS'):12:3 C.L('KOOTEN'):12:3/;

  PUT storin;
  PUT S5('PEACE',U):12:3 S5('CENTRAL',U):12:3 S5('CARIBOO',U):12:3
      S5('THOMPS',U):12:3 S5('KOOTEN',U):12:3/;

  PUT tostor;
  PUT PRICE('STORAGE'):12:3 TR.L('STORAGE','PEACE'):12:3
      TR.L('STORAGE','CENTRAL'):12:3 TR.L('STORAGE','CARIBOO'):12:3
      TR.L('STORAGE','THOMPS'):12:3 TR.L('STORAGE','KOOTEN'):12:3/;

*****END OF LOOP 6
  );
```

```
*Shazam command file for generating the variance / covariance matrix for
*forage production and Cholesky decomposed matrix (from the coef. of
*variation / correlation coef. matrix from precipitation data)
*
file 4 cholreg.dat
file 6 cholreg.out
sample 1 1
read(4) M1 M2 M3 M4 M5
*
* k1 and k2 are coefficients which relate variability in rainfall to
* variability in production
*
genr k1=0.3
genr k2=0.3
*
* vi=coefficient of variation
* vi(prod)=k1*Si(rain)/Mi(rain)=Si(prod)/Mi(prod)
genr v1=k1*0.5894
genr v2=k1*0.4350
genr v3=k1*0.5165
genr v4=k1*0.4878
genr v5=k1*0.4522
* We know that Si(prod)=vi(prod)*Mi(prod)
* ROWij=k2*COVij(rain)/Si(rain)*Sj(rain)=COVij(prod)/Si(prod)*Sj(prod)
* therefore, COVij(prod)=ROWij*Si(prod)*Sj(prod)
*
* For diagnol elements of covariance matrix (i.e. own region variance)
*
* ROWij=coefficient of correlation
genr ROW12=k2*0.5450
genr ROW13=k2*0.4581
genr ROW14=k2*0.3634
genr ROW15=k2*0.2957
genr ROW23=k2*0.8297
genr ROW24=k2*0.5259
genr ROW25=k2*0.3698
genr ROW34=k2*0.6207
genr ROW35=k2*0.4373
genr ROW45=k2*0.6528
* (ROWij=COV(Qi,Qj)/SQRT(SIGMAii)*SQRT(SIGMAjj))
* COV(Qi,Qj)=ROWij*SIGMAi*SIGMAj
*
* Creating elements of covariance matrix
genr S1=v1*M1
genr S11=S1**2
genr S2=v2*M2
genr S22=S2**2
genr S3=v3*M3
genr S33=S3**2
genr S4=v4*M4
genr S44=S4**2
genr S5=v5*M5
genr S55=S5**2
genr S12=ROW12*(S1*S2)
genr S21=S12
genr S13=ROW13*(S1*S3)
genr S31=S13
genr S14=ROW14*(S1*S4)
genr S41=S14
genr S15=ROW15*(S1*S5)
genr S51=S15
genr S23=ROW23*(S2*S3)
genr S32=S23
genr S24=ROW24*(S2*S4)
genr S42=S24
genr S25=ROW25*(S2*S5)
```

```
genr S52=S25
genr S34=ROW34*(S3*S4)
genr S43=S34
genr S35=ROW35*(S3*S5)
genr S53=S35
genr S45=ROW45*(S4*S5)
genr S54=S45
*
*1st column of cov matrix
matrix S1=S11|S21|S31|S41|S51
print S1
matrix S2=S12|S22|S32|S42|S52
matrix S3=S13|S23|S33|S43|S53
matrix S4=S14|S24|S34|S44|S54
matrix S5=S15|S25|S35|S45|S55
matrix Covmat=S1'|S2'|S3'|S4'|S5'
*The manually generated covariance matrix
print covmat
*
Matrix p=CHOL(covmat)
*THE CHOLESKY DECOMPOSITION MATRIX
print p
stop
```